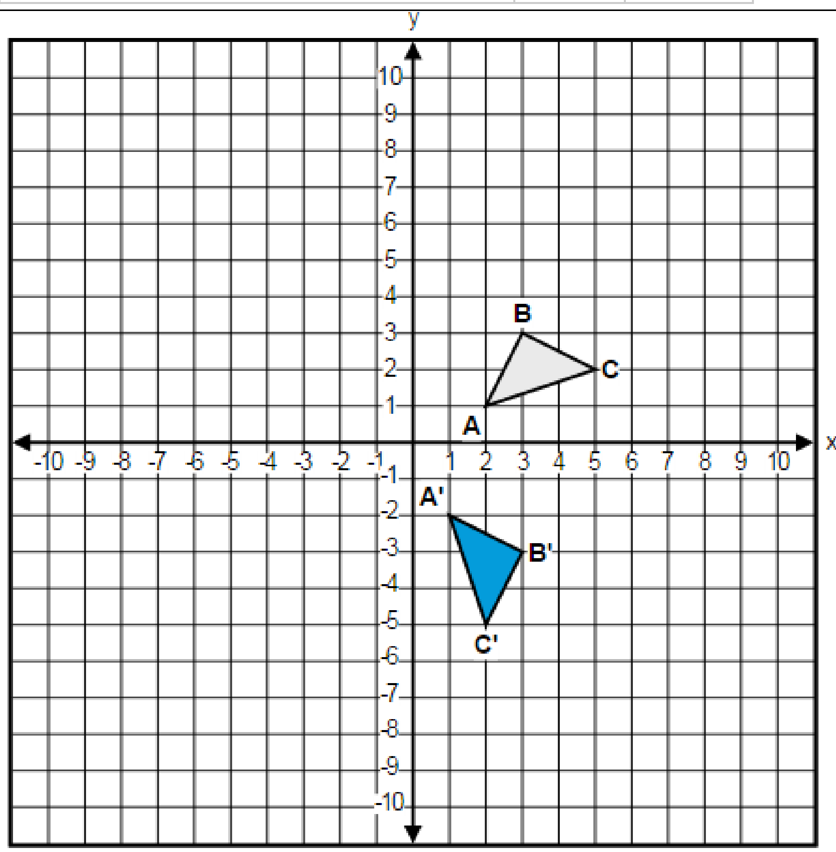
PARCC
Partnership for Assessment of Readiness for College and Careers

Released Test Answer and Alignment Document

Mathematics – Geometry

Spring 2016

<table>
<tr><th>Item Number</th><th>Answer Key</th><th>Evidence Statement Key</th><th>Integrated Course Alignment</th></tr>
<tr><td>1.</td><td>
<table>
<tr><th></th><th>Correct</th><th>Incorrect</th></tr>
<tr><td>If two lines in the same plane do not intersect, the lines must be parallel.</td><td>☑</td><td>☐</td></tr>
<tr><td>If two lines in space do not intersect, the lines must be parallel.</td><td>☐</td><td>☑</td></tr>
<tr><td>If two lines are parallel, the lines must lie in the same plane.</td><td>☑</td><td>☐</td></tr>
</table>
</td><td>G-CO.1</td><td>Math 1</td></tr>
<tr><td>2.</td><td>
</td><td>G-CO.6</td><td>Math 1</td></tr>
<tr><td>3.</td><td>C</td><td>G-SRT.1b</td><td>Math 2</td></tr>
</table>

4.	A	G-SRT.1b	Math 2
5.	A, D	G-SRT.6	Math 2
6.	The coordinates of point Q are (6 , 0).	G-GPE.6	Math 3
7.	B	G-GMD.4	Math 3
8.	2	G-GMD.3	Math 2
9.	Part A: B Part B: D	G-GPE.1-2	Math 3
10.	B	G-CO.5	Math 1
11.	B, C, D	G-CO.6	Math 1
12.	Side $A'B'$ will be parallel to side AB. Side $A'C'$ will be parallel to side AC. Side $B'C'$ will lie on the same line as side BC.	G-SRT.1a	Math 2
13.	A, C, D, E	G-SRT.1a	Math 2
14.	A, C, D	G-SRT.2	Math 2
15.	B, D, G	G-SRT.7-2	Math 2
16.	14 or 15	G-SRT.8	Math 2
17.	C	G-GPE.6	Math 3
18.	B, E, H	G-GMD.1	Math 2
19.	B	G-C.2	Math 3
20.	C	G-SRT.5	Math 2
21.	Part A: 87 Part B: 99	G-SRT.8	Math 2
22.	Part A: A, B, D Part B: D	G-CO.D	Math 3

23.	Part A: Draw arcs with the same radius, centered at *P*, that intersect $\overrightarrow{PY}$ and $\overrightarrow{PA}$. Draw two intersecting arcs with the same radius, one centered at *Y* and one centered at *A*, on the interior of $\angle P$. Name the intersection of the two arcs *X*. Using a straightedge, draw the line through *P* and *X*. First Last Part B: The angle bisector of $\angle P$ is $\overrightarrow{PX}$ because $\triangle YPX$ is congruent to $\triangle APX$ by [SSS ▾]. Therefore $\angle YPX$ is congruent to $\angle APX$ because they are corresponding angles of congruent triangles.	G-CO.D	Math 3
24.	Part A: A, C, E Part B: D	G-C.2	Math 3
25.	Part A: A Part B: D	G-Int.1	Math 3
26.	Part A: C Part B: B Part C: A, C, E Part D: C	G-SRT.8	Math 2
27.	See Rubric	HS-C.13.2	Math 3
28.	See Rubric	HS-C.18.2	
29.	Part A: See Rubric Part B: See Rubric Part C: See Rubric	HS-D.1-2	Math 2
30.	Part A: See Rubric Part B: See Rubric	HS-D.2-11	Math 2

#27 Rubric	
Score	**Description**
3	Student response includes the following 3 elements. • **Reasoning component** = 2 points o Determination that the figure is a parallelogram o Valid explanation of equal lengths for pairs of opposite sides or valid explanation of parallel sides • **Computation component** = 1 point o Correct computation of slopes or lengths Sample Student Response: A four-sided figure with opposite sides parallel meets the conditions for a parallelogram. The side $\overline{OT}$ lies on the x-axis, which is horizontal. Therefore, its slope is 0. Side $\overline{PS}$ also lies on a horizontal line because each endpoint has the same *y*-coordinate. Therefore, it also has slope 0. Because the two sides have the same slope, they must be parallel. The side $\overline{OP}$ lies on a line with slope $\frac{b-0}{a-0} = \frac{b}{a}$. Side $\overline{TS}$ lies on a line with slope $\frac{b-0}{a+c-c} = \frac{b}{a}$. Because both sides have the same slope, they are parallel. Therefore, the figure is a parallelogram. OR: A four-sided figure with opposite sides the same length meets the conditions for a parallelogram. The endpoints of side $\overline{OT}$ have the same *y*-coordinate, so its length is the difference of the *x*-coordinates, $c - 0 = c$The endpoints of side $\overline{PS}$ have the same *y*-coordinate, so its length is the difference of the *x*-coordinates, $a + c - a = c$.Therefore, opposite sides have the same length. The length of side $\overline{OP}$ is $\sqrt{(a-0)^2+(b-0)^2} = \sqrt{a^2+b^2}$, found by using the distance formula. (Note: student could use a right triangle argument). The length of side *TS* is $\sqrt{(a+c-c)^2+(b-0)^2} = \sqrt{a^2+b^2}$. Opposite sides have the same length therefore; the figure is a parallelogram.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

<table>
<tr><th colspan="2">#28 Rubric</th></tr>
<tr><th>Score</th><th>Description</th></tr>
<tr><td>4</td><td>Student response includes the following 4 elements.

Computation component = 1 point
Correct value for x: −6
Reasoning component = 3 points

Establishment of similar triangles by assuming lines are parallel
Correct work shown for solving for x using the given side lengths and proportionality
Recognition that length cannot be negative and that the dimensions are not reasonable.

Sample Student Response:

If $\overline{AB}$ is parallel to $\overline{DE}$ than $\angle B \cong \angle E$ and $\angle A \cong \angle D$, since both pairs of angles are alternate-interior angles formed by the parallel lines and the transversals $\overline{BE}$ and $\overline{AD}$, respectively. Because two angles in triangle *ABC* are congruent to two angles in triangle *CDE*, the triangles *ABC* and *DEC* are similar.
Because the lengths of corresponding sides of similar triangles are in proportion $\frac{AC}{CD} = \frac{BC}{CE}$, and by substituting the given values,

$$\frac{x}{x+2} = \frac{x+3}{x+4}$$
$$x(x+4) = (x+2)(x+3)$$
$$x^2 + 4x = x^2 + 5x + 6$$
$$-6 = x$$
The length of one side cannot be negative, so these dimensions are not reasonable.

Note: Alternative methods are possible and any valid method is acceptable for full credit.</td></tr>
<tr><td>3</td><td>Student response includes 3 of the 4 elements.</td></tr>
<tr><td>2</td><td>Student response includes 2 of the 4 elements.</td></tr>
<tr><td>1</td><td>Student response includes 1 of the 4 elements.</td></tr>
<tr><td>0</td><td>Student response is incorrect or irrelevant.</td></tr>
</table>

#29 Rubric Part A	
Score	**Description**

2	Student response includes the following 2 elements.
	
	• **Computation component** = 1 point
	  o Correct volume of 384 cubic inches for the stones.
	
	• **Modeling component** = 1 point
	  o Correct work to support answer.
	
	Sample Student Response:
	
	Because they are contained within the aquarium, the water and the combination of water and stones each have the shape of a rectangular prism. The formula for the volume V of a rectangular prism is $V = lwh$, where l is the length of the prism, w is the width, and h is the height.
	
	Volume (stones) = Volume (stones + water) – Volume (water)
	
	Volume (stones + water) = 16 in. × 8 in. × 7 in. = 896 in^3
	
	Volume (water) = 16 in. × 8 in. × 4 in. = 512 in^3
	
	Volume (stones) = 896 in^3 - 512 in^3 = 384 in^3
	
	So the volume of the stones is 384 cubic inches.

1	Student response includes 1 of the 2 elements.
0	Student response is incorrect or irrelevant.

#29 Rubric Part B	
Score	**Description**

3	Student response includes the following 4 elements.
	
	• **Computation component** = 2 points
	  o Correct volume of 343 cubic inches for the cube.
	  o Correct volume of 200π (or ≈628) cubic inches for the cylinder.
	  o Correct volume of $\frac{325\pi}{3}$ (or ≈340) cubic inches for the cone.
	
	• **Modeling component** = 1 point
	  o Correct work to support each volume calculation.

	Sample Student Response: <u>Volume of the cube</u> Use the formula for the volume, *V*, of a cube. $V = s^3$ $V = 7^3$ $V = 343$ (cubic inches) <u>Volume of the cylinder</u> Use the area formula for a circle to find the area, *B*, of the base. $C = \pi r^2$ $B = \pi(5)^2$ $B = 25\pi$ (square inches) Use the formula for the volume, *V*, of a cylinder. $V = Bh$ $V = (25\pi)(8) = 200\pi$ $V \approx 628$ (cubic inches) <u>Volume of the cone</u> Use the area formula for a circle to find the area, *B,* of the base. $C = \pi r^2$ $B = \pi(5)^2$ $B = 25\pi$ (square inches) Use the formula for the volume, *V,* of a cone. $V = \frac{Bh}{3} = \frac{(25\pi)(13)}{3} = \frac{325\pi}{3}$ $V \approx 340$ (cubic inches)
2	Student response includes 3 of the 4 elements.
1	Student response includes 1-2 of the 4 elements.
0	Student response is incorrect or irrelevant.

#29 Rubric Part C	
Score	**Description**
1	Student response includes the following element. • **Modeling component** = 1 point o Choice of the cylinder and a logical explanation for the choice. Sample Student Response: The stones have irregular shapes, so there will be some empty space between them when they are placed in the container. This means that the volume of the container must be a bit greater than 384 cubic inches. The volumes of the cube and the cone are less than the volume of the stones, so the stones will not fit inside either of these shapes. The volume of the cylinder is more than $1\frac{1}{2}$ times the volume of the stones ($384 \times 1\frac{1}{2} = 576$ and $628 > 576$). The stones will fit inside the cylinder.
0	Student response is incorrect or irrelevant.

#30 Rubric Part A	
Score	**Description**

2	Student response includes the following 2 elements.
	• **Modeling component** = 1 point
	  o Valid explanation or work to calculate the height of the support
	• **Computation component** = 1 point
	  o Correct height of the support at 1.7 feet
	
	Sample Student Response:
	
	Let *x* represent the height of the support. A right angle is formed with a 25° angle and a hypotenuse of 4. A possible equation and solution:
	$\frac{x}{4} = \sin 25°$
	$x = 4\sin 25°$
	$x \approx 1.7\ ft$

1	Student response includes 1 of the 2 elements.

0	Student response is incorrect or irrelevant.

<table>
<tr><th colspan="2">#30 Rubric Part B</th></tr>
<tr><th>Score</th><th>Description</th></tr>
<tr><td>1</td><td>Student response includes the following element.
Modeling component = 1 point
Valid model and height for Point Q.
Sample Student Response:

I can draw a line continuation of line segment QS from point Q to the ground creating a right triangle. The distance from point Q to where the hypotenuse of the right triangle touches the ground can be represented as y. Therefore, the hypotenuse from point R to the ground could be represented by 4 + y. I can then find y as follows:
$\cos 80° = \frac{1.7}{4+y}$
$4 + y = \frac{1.7}{\cos 80°}$
$4 + y \approx 9.79$
$y \approx 5.79$

From there, I will let the distance from point Q to the ground be represented by z. I can find the length of that segment as follows:
$\cos 80° = \frac{z}{5.79}$
$x = 5.79(\cos 80°)$
$z \approx 1.005$

Therefore, the distance from point Q to the ground is approximately 1.0 foot.

Or:

The angle created by the seating board and the left side of the central support is 80°. I can draw a perpendicular line from point Q to the central support, RT, creating a right triangle. The distance from point Q to the ground is the same as the distance from the newly drawn line to the ground. Let y represent that distance. Then the distance along the central support from the drawn line to point T can be represented by 1.7 – y.</td></tr>
</table>

	$\cos 80^\circ = \frac{1.7 - y}{4}$ $4\cos 80^\circ = 1.7 - y$ $y = 1.7 - 4\cos 80^\circ$ $y \approx 1.0054$ $y \approx 1.0\ ft$ Therefore, the distance from point Q to the ground is approximately 1.0 foot. **Note:** Without support, an answer of 1 foot does not earn any credit. A logical explanation of how to arrive at the height of Point Q from the ground with the correct answer of 1 foot is necessary to earn the point for part B. The modeling of setup and work needs to show understanding of the process, but may contain some vague statements or minor errors.
0	Student response is incorrect or irrelevant.