

# PARCC MODEL CONTENT FRAMEWORKS

MATHEMATICS

GEOMETRY

Version 3.0

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## PARCC MODEL CONTENT FRAMEWORK FOR MATHEMATICS FOR GEOMETRY

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### Geometry Overview

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. For more information, see Tables 1 and 2. Course emphases are indicated by: ■ Major Content; □ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

#### Congruence (G-CO)

- Experiment with transformations in the plane (1, 2, 3, 4, 5)
- Understand congruence in terms of rigid motions (6, 7, 8)
- Prove geometric theorems (9, 10, 11)
- Make geometric constructions (12, 13)

#### Similarity, Right Triangles, and Trigonometry (G-SRT)

- Understand similarity in terms of similarity transformations (1, 2, 3)
- Prove theorems using similarity (4, 5)
- Define trigonometric ratios and solve problems involving right triangles (6, 7, 8)

#### Circles (G-C)

- Understand and apply theorems about circles (1, 2, 3)
- Find arc lengths and areas of sectors of circles (5)

#### Expressing Geometric Properties with Equations (G-GPE)

- Translate between the geometric description and the equation of a conic section (1)
- Use coordinates to prove simple geometric theorems algebraically (4, 5, 6, 7)

#### Geometric measurement and dimension (G-GMD)

- Explain volume formulas and use them to solve problems (1, 3)
- Visualize relationships between two-dimensional and three-dimensional objects (4)

#### Modeling with Geometry (G-MG)

- Apply geometric concepts in modeling situations (1, 2, 3)

#### Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Examples of Key Advances from Previous Grades or Courses

- Because concepts such as rotation, reflection, and translation were treated in the grade 8 standards mostly in the context of hands-on activities, and with an emphasis on geometric intuition, high school Geometry will put equal weight on precise definitions.
- In grades K-8, students worked with a variety of geometric measures (length, area, volume, angle, surface area, and circumference). In high school Geometry, students apply these component skills in tandem with others in the course of modeling tasks and other substantial applications (MP.4).
- The skills that students develop in Algebra I around simplifying and transforming square roots will be useful when solving problems that involve distance or area and that make use the Pythagorean theorem.
- In grade 8, students learned the Pythagorean theorem and used it to determine distances in a coordinate system (8.G.6–8). In high school Geometry, students will build on their understanding of distance in coordinate systems and draw on their growing command of algebra to connect equations and graphs of circles (G-GPE.1).
- The algebraic techniques developed in Algebra I can be applied to study analytic geometry. Geometric objects can be analyzed by the algebraic equations that give rise to them. Some basic geometric theorems in the Cartesian plane can be proven using algebra.

## Discussion of Mathematical Practices in Relation to Course Content

- **Reason abstractly and quantitatively** (MP.2). Abstraction is used in geometry when, for example, students use a diagram of a specific isosceles triangle as an aid to reason about *all* isosceles triangles (G-CO.9). Quantitative reasoning in geometry involves the real numbers in an essential way: Irrational numbers show up in work with the Pythagorean theorem (G-SRT.8), area formulas often depend (subtly and informally) on passing to the limit and real numbers are an essential part of the definition of dilation (G-SRT.1). The proper use of units can help students understand the effect of dilation on area and perimeter (N-Q.1).
- **Construct viable arguments and critique the reasoning of others** (MP.3). While all of high school mathematics should work to help students see the importance and usefulness of deductive arguments, geometry is an ideal arena for developing the skill of creating and presenting proofs (G-CO.9.10). One reason is that conjectures about geometric phenomena are often about infinitely many cases at once — for example, *every* angle inscribed in a semicircle is a right angle — so that such results cannot be established by checking every case (G-C.2).
- **Use appropriate tools strategically** (MP.5). Dynamic geometry environments can help students look for invariants in a whole class of geometric constructions, and the constructions in such environments can sometimes lead to an idea behind a proof of a conjecture.
- **Attend to precision** (MP.6). Teachers might use the activity of creating definitions as a way to help students see the value of precision. While this is possible in every course, the activity has a particularly visual appeal in geometry. For example, a class can build the definition of *quadrilateral* by starting with a rough idea (“four sides”), gradually refining the idea so that it rules out figures that do not fit the intuitive idea. Another place in geometry where precision is

necessary and useful is in the refinement of conjectures so that initial conjectures that are not correct can be salvaged — two angle measures and a side length do not determine a triangle, but a certain configuration of these parts leads to the angle-side-angle theorem (G-CO.8).

- **Look for and make use of structure** (MP.7). Seeing structure in geometric configurations can lead to insights and proofs. This often involves the creation of auxiliary lines not originally part of a given figure. Two classic examples are the construction of a line through a vertex of a triangle parallel to the opposite side as a way to see that the angle measures of a triangle add to 180 degrees and the introduction of a symmetry line in an isosceles triangle to see that the base angles are congruent (G-CO.9, 10). Another kind of hidden structure makes use of area as a device to establish results about proportions, such as the important theorem (and its converse) that a line parallel to one side of a triangle divides the other two sides proportionally (G-SRT.4).

## Fluency Recommendations

- G-SRT.5** Fluency with the triangle congruence and similarity criteria will help students throughout their investigations of triangles, quadrilaterals, circles, parallelism, and trigonometric ratios. These criteria are necessary tools in many geometric modeling tasks.
- G-GPE.4, 5, 7** Fluency with the use of coordinates to establish geometric results, calculate length and angle, and use geometric representations as a modeling tool are some of the most valuable tools in mathematics and related fields.
- G-CO.12** Fluency with the use of construction tools, physical and computational, helps students draft a model of a geometric phenomenon and can lead to conjectures and proofs.

## Pathway Summary Table: AI – G – All Pathway

**Table 1.** This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course. Algebra I and II are adjacent so as to make the shading continuous, despite being generally taught a year apart.

		CCSSM Standard	A I	A II	G
Number & Quantity		N-RN.1		▪	
		N-RN.2		▪	
		N-RN.3	▪		
		N-Q.1	▪		
		N-Q.2	▪	▪	
		N-Q.3	▪		
		N-CN.1		▪	
		N-CN.2		▪	
		N-CN.7		▪	
		A-SSE.1	▪		
		A-SSE.2	▪	▪	
		A-SSE.3a	▪		
A-SSE.3b	▪				
A-SSE.3c	▪	▪			
A-SSE.4		▪			
A-APR.1	▪				
A-APR.2		▪			
A-APR.3	▪	▪			
A-APR.4		▪			
A-APR.6		▪			
A-CED.1	▪	▪			
A-CED.2	▪				
A-CED.3	▪				
A-CED.4	▪				
A-REI.1	▪	▪			
A-REI.2		▪			
A-REI.3	▪				
A-REI.4a	▪				
A-REI.4b	▪	▪			
A-REI.5	▪				
A-REI.6	▪	▪			
A-REI.7		▪			
A-REI.10	▪				
A-REI.11	▪	▪			
A-REI.12	▪				
Algebra		F-IF.1	▪		
		F-IF.2	▪		
		F-IF.3	▪	▪	
		F-IF.4	▪	▪	
		F-IF.5	▪		
		F-IF.6	▪	▪	
		F-IF.7a	▪		
		F-IF.7b	▪		
		F-IF.7c		▪	
		F-IF.7e		▪	
		F-IF.8a	▪		
		F-IF.8b		▪	
		F-IF.9	▪	▪	
		F-BF.1a	▪	▪	
		F-BF.1b		▪	
		F-BF.2		▪	
		F-BF.3	▪	▪	
		F-BF.4a		▪	
		F-LE.1	▪		
		F-LE.2	▪	▪	
		F-LE.3	▪		
		F-LE.4		▪	
		F-LE.5	▪	▪	
		F-TF.1		▪	
F-TF.2		▪			
F-TF.5		▪			
F-TF.8		▪			
Geometry		G-CO.1			▪
		G-CO.2			▪
		G-CO.3			▪
		G-CO.4			▪
		G-CO.5			▪
		G-CO.6			▪
		G-CO.7			▪
		G-CO.8			▪
		G-CO.9			▪
		G-CO.10			▪
		G-CO.11			▪
		G-CO.12			▪
		G-CO.13			▪
		G-SRT.1			▪
		G-SRT.2			▪
		G-SRT.3			▪
		G-SRT.4			▪
		G-SRT.5			▪
		G-SRT.6			▪
		G-SRT.7			▪
		G-SRT.8			▪
		G-C.1			▪
		G-C.2			▪
		G-C.3			▪
		G-C.5			▪
		G-GPE.1			▪
		G-GPE.2		▪	
		G-GPE.4			▪
		G-GPE.5			▪
		G-GPE.6			▪
		G-GPE.7			▪
		G-GMD.1			▪
		G-GMD.3			▪
		G-GMD.4			▪
		G-MG.1			▪
		G-MG.2			▪
G-MG.3			▪		
Statistics		S-ID.1	▪		
		S-ID.2	▪		
		S-ID.3	▪		
		S-ID.4		▪	
		S-ID.5	▪		
		S-ID.6a	▪	▪	
		S-ID.6b	▪		
		S-ID.6c		▪	
		S-ID.7	▪		
		S-ID.8	▪		
		S-ID.9	▪		
		S-IC.1		▪	
		S-IC.2		▪	
		S-IC.3		▪	
		S-IC.4		▪	
		S-IC.5		▪	
		S-IC.6		▪	
		S-CP.1		▪	
		S-CP.2		▪	
		S-CP.3		▪	
		S-CP.4		▪	
		S-CP.5		▪	
		S-CP.6		▪	
		S-CP.7		▪	

## Assessment Limits for Standards Assessed on More Than One End-of-Course Test: AI-G-AII Pathway

**Table 2.** This draft table shows assessment limits for standards assessed on more than one end-of-course test. (These “cross-cutting” standards are visible as shaded cells in Table 1.)

CCSSM Cluster	CCSSM Key	CCSSM Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
<b>Reason quantitatively and use units to solve problems</b>	N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.	This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.
<b>Interpret the structure of expressions</b>	A-SSE.2	Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i>	<p>i) Tasks are limited to numerical expressions and polynomial expressions in one variable.</p> <p>ii) Examples: Recognize <math>53^2 - 47^2</math> as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form <math>(53+47)(53-47)</math>. See an opportunity to rewrite <math>a^2 + 9a + 14</math> as <math>(a+7)(a+2)</math>.</p>	<p>i) Tasks are limited to polynomial, rational, or exponential expressions.</p> <p>ii) Examples: see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>. In the equation <math>x^2 + 2x + 1 + y^2 = 9</math>, see an opportunity to rewrite the first three terms as <math>(x+1)^2</math>, thus recognizing the equation of a circle with radius 3 and center <math>(-1, 0)</math>. See <math>(x^2 + 4)/(x^2 + 3)</math> as <math>((x^2+3) + 1)/(x^2+3)</math>, thus recognizing an opportunity to write it as <math>1 + 1/(x^2 + 3)</math>.</p>
<b>Write expressions in equivalent forms to solve problems</b>	A-SSE.3c	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. <sup>62</sup> (c) Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15^t</math> can be rewritten as <math>(1.15^{1/12})^{12t} \approx 1.012^{12t}</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i>	<p>i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.</p> <p>ii) Tasks are limited to exponential expressions with integer exponents.</p>	<p>i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation.</p> <p>ii) Tasks are limited to exponential expressions with rational or real exponents.</p>
<b>Understand the relationship between zeros and factors of polynomials</b>	A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	i) Tasks are limited to quadratic and cubic polynomials in which linear and quadratic factors are available. For example, find the zeros of $(x - 2)(x^2 - 9)$ .	i) Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$

CCSSM Cluster	CCSSM Key	CCSSM Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
<b>Create equations that describe numbers or relationships</b>	A-CED.1	Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>	i) Tasks are limited to linear, quadratic, or exponential equations with integer exponents.	i) Tasks are limited to exponential equations with rational or real exponents and rational functions.  ii) Tasks have a real-world context.
<b>Understand solving equations as a process of reasoning and explain the reasoning</b>	A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	i) Tasks are limited to quadratic equations.	i) Tasks are limited to simple rational or radical equations.
<b>Solve equations and inequalities in one variable</b>	A-REI.4b	Solve quadratic equations in one variable. b) Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .	i) Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions.  <i>Note, solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials (cluster A-APR.B). Cluster A-APR.B is formally assessed in A2.</i>	i) In the case of equations that have roots with nonzero imaginary parts, students write the solutions as $a \pm bi$ for real numbers $a$ and $b$ .
<b>Solve systems of equations</b>	A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	i) Tasks have a real-world context. ii) Tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	i) Tasks are limited to 3x3 systems.
<b>Represent and solve equations and inequalities graphically</b>	A-REI.11	Explain why the $x$ -coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. *	i) Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. ii) Finding the solutions approximately is limited to cases where $f(x)$ and $g(x)$ are polynomial functions.	i) Tasks may involve any of the function types mentioned in the standard.
<b>Understand the concept of a function and use function notation</b>	F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i>	i) This standard is part of the Major work in Algebra I and will be assessed accordingly.	i) This standard is Supporting work in Algebra II. This standard should support the Major work in F-BF.2 for coherence.

CCSSM Cluster	CCSSM Key	CCSSM Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
<b>Interpret functions that arise in applications in terms of a context</b>	F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> *	<p>i) Tasks have a real-world context.</p> <p>ii) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.</p> <p><i>Compare note (ii) with standard F-IF.7.</i></p> <p><i>The function types listed here are the same as those listed in the Algebra I column for standards F-IF.6 and F-IF.9.</i></p>	<p>i) Tasks have a real-world context</p> <p>ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.</p> <p><i>Compare note (ii) with standard F-IF.7.</i></p> <p><i>The function types listed here are the same as those listed in the Algebra II column for standards F-IF.6 and F-IF.9.</i></p>
<b>Interpret functions that arise in applications in terms of a context</b>	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *	<p>i) Tasks have a real-world context.</p> <p>ii) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.</p> <p><i>The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.9.</i></p>	<p>i) Tasks have a real-world context.</p> <p>ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.</p> <p><i>The function types listed here are the same as those listed in the Algebra II column for standards F-IF.4 and F-IF.9.</i></p>
<b>Analyze functions using different representations</b>	F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.) <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i>	<p>i) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.</p> <p><i>The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.6.</i></p>	<p>i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.</p> <p><i>The function types listed here are the same as those listed in the Algebra II column for standards F-IF.4 and F-IF.6.</i></p>
<b>Build a function that models a relationship between two quantities</b>	F-BF.1a	Write a function that describes a relationship between two quantities.* a) Determine an explicit expression, a recursive process, or steps for calculation from a context.	<p>i) Tasks have a real-world context.</p> <p>ii) Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.</p>	<p>i) Tasks have a real-world context</p> <p>ii) Tasks may involve linear functions, quadratic functions, and exponential functions.</p>
<b>Build new functions from</b>	F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x+k)$ for specific values of $k$ (both positive and negative); find the value of	<p>i) Identifying the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x+k)</math> for specific values of <math>k</math> (both positive and negative) is limited</p>	<p>i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions</p> <p>ii) Tasks may involve recognizing even and odd</p>

CCSSM Cluster	CCSSM Key	CCSSM Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
existing functions		$k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i>	<p>to linear and quadratic functions.</p> <p>ii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.</p> <p>iii) Tasks do not involve recognizing even and odd functions.</p> <p><i>The function types listed in note (ii) are the same as those listed in the Algebra I column for standards F-IF.4, F-IF.6, and F-IF.9.</i></p>	<p>functions.</p> <p><i>The function types listed in note (i) are the same as those listed in the Algebra II column for standards F-IF.4, F-IF.6, and F-IF.9.</i></p>
Construct and compare linear, quadratic, and exponential models and solve problems	F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	i) Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).	i) Tasks will include solving multi-step problems by constructing linear and exponential functions.
Interpret expressions for functions in terms of the situation they model	F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.	<p>i) Tasks have a real-world context.</p> <p>ii) Exponential functions are limited to those with domains in the integers.</p>	<p>i) Tasks have a real-world context.</p> <p>ii) Tasks are limited to exponential functions with domains not in the integers.</p>
Summarize, represent, and interpret data on two categorical and quantitative variables	S-ID.6a	<p>Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p>a) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</i></p>	<p>i) Tasks have a real-world context.</p> <p>ii) Exponential functions are limited to those with domains in the integers.</p>	<p>i) Tasks have a real-world context.</p> <p>ii) Tasks are limited to exponential functions with domains not in the integers and trigonometric functions.</p>