Resource Materials for Teachers

2012-2013
NMSBA/HIGH SCHOOL GRADUATION ASSESSMENT (HSGA)

Grade 11 MATH Items for Practice
ACKNOWLEDGEMENT

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Resource Materials for Teachers

This teacher resource document accompanies the “NMSBA/HSGA MATH Items for Practice” student booklet; it contains an answer key for practice items and student samples of short answer responses with corresponding scoring guides.

Contents of Student Booklet

The “NMSBA/HSGA MATH Items for Practice” student booklet contains 16 math practice items that are representative of test items that may appear on the spring 2013 graduation exit exam. These practice items are intended to help students prepare for the actual NMSBA/HSGA.

Specifically, the student booklet contains 10 multiple-choice (MC) questions, 4 short answer (SA) response questions, and 2 constructed response (CR) questions. Each MC question has four possible answers; only one of the four choices is the correct answer. The SA and CR questions require written responses.

Usage of Student Booklet

Teachers are encouraged to use the student booklet with math practice items to help students prepare for the NMSBA/HSGA. Teachers can administer the student booklet in its entirety during one sitting, and then after grading, critically review all item answers with students. Alternatively, teachers can take a selection of questions at a time and critically review item answers orally with students in a large group setting. The student booklet and answer sheet (for MC practice items) can be found in Data Director (www.achievedata.com/aps) under Assessments using the following search criteria:

<table>
<thead>
<tr>
<th>Subject</th>
<th>Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>2012-2013</td>
</tr>
<tr>
<td>Grade Level</td>
<td>11</td>
</tr>
<tr>
<td>Test Type</td>
<td>District</td>
</tr>
<tr>
<td>Search by Assessment Title</td>
<td>2012-2013 NMSBA/HSGA Math Practice Items</td>
</tr>
<tr>
<td></td>
<td>2012-2013 NMSBA/HSGA Math Practice Items (SPANISH)</td>
</tr>
</tbody>
</table>

Questions?

This teacher resource document is also located in Data Director (use same search criteria as above). If you have any questions about either this document or the “NMSBA/HSGA MATH Items for Practice” student booklet, contact:

- Dr. Donna Navarrete, RDA secondary assessment resource teacher (navarrete_d@aps.edu, 505.872.6866); or
- Dr. Mike Loughrey, RDA assessment manager (loughrey@aps.edu, 505.872.6822).
# Answer Key

<table>
<thead>
<tr>
<th>Item</th>
<th>Assessment Target(^1)</th>
<th>DOK(^2)</th>
<th>Item Type(^3)</th>
<th>Answer</th>
<th>Points Possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FA</td>
<td>1</td>
<td>MC</td>
<td>D</td>
<td>1</td>
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<tr>
<td>2</td>
<td>GM</td>
<td>2</td>
<td>MC</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>FA</td>
<td>2</td>
<td>MC</td>
<td>D</td>
<td>1</td>
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<tr>
<td>4</td>
<td>GM</td>
<td>2</td>
<td>MC</td>
<td>A</td>
<td>1</td>
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<tr>
<td>5</td>
<td>GM</td>
<td>2</td>
<td>SA</td>
<td>*</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>GM</td>
<td>2</td>
<td>CR</td>
<td>*</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>GM</td>
<td>2</td>
<td>MC</td>
<td>A</td>
<td>1</td>
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<tr>
<td>8</td>
<td>FA</td>
<td>1</td>
<td>MC</td>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>DSP</td>
<td>2</td>
<td>MC</td>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>FA</td>
<td>2</td>
<td>MC</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>FA</td>
<td>2</td>
<td>SA</td>
<td>*</td>
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<tr>
<td>12</td>
<td>DSP</td>
<td>2</td>
<td>SA</td>
<td>*</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>DSP</td>
<td>3</td>
<td>SA</td>
<td>*</td>
<td>2</td>
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<tr>
<td>14</td>
<td>GM</td>
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<td>MC</td>
<td>B</td>
<td>1</td>
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<tr>
<td>15</td>
<td>GM</td>
<td>2</td>
<td>MC</td>
<td>B</td>
<td>1</td>
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<tr>
<td>16</td>
<td>GM</td>
<td>3</td>
<td>CR</td>
<td>*</td>
<td>4</td>
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</tbody>
</table>

\(^1\)FA = Functions & Algebra, GM = Geometry & Measurement, DSP = Data, Statistics, & Probability.  
\(^2\)Depth of Knowledge (DOK): 1 = recall & reproduction, 2 = skills & concepts, 3 = strategic thinking.  
\(^3\)Item Type: MC = Multiple Choice, SA = Short Answer, CR = Constructed Response

*Scoring guide for item (with samples of student work) is provided on the following pages.*
5. Look at parallelogram $EFGH$.

The diagonals of parallelogram $EFGH$ intersect at point $P$. What are the coordinates of point $P$?

**Scoring Guide**

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student gives correct coordinates, $\left(3, \frac{1}{2}\right)$</td>
</tr>
<tr>
<td>0</td>
<td>Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.</td>
</tr>
<tr>
<td>Blank</td>
<td>No response</td>
</tr>
</tbody>
</table>
Sample 1-Point Score

5.

\[ \frac{2}{4} = 0.5 \]

\[ \frac{7}{6} - 3 \]

\[ \frac{1}{1} = 0.5 \]

\[ (3, 0.5) \]

Sample 1-Point Score

5.

\( (3, 0.5) \)

Sample 0-Point Score

5.

The coordinates of point \( P \) is

\( (3, 1) \)
6. Triangle $KLM$ is similar to triangle $GHJ$ ($\triangle KLM \sim \triangle GHJ$).

The perimeter of triangle $KLM$ is 16 centimeters. The perimeter of triangle $GHJ$ is 40 centimeters.

a. What is the ratio of side $\overline{ML}$ to side $\overline{JH}$?

b. What is the ratio of the area of triangle $KLM$ to the area of triangle $GHJ$?

### Scoring Guide:

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>for correct side ratio, 2 to 5 or equivalent, and correct area ratio, 4 to 25 or equivalent</td>
</tr>
<tr>
<td>1</td>
<td>for correct side ratio but not correct area ratio OR for correct area ratio but not correct side ratio OR for correct solution to part b based on an incorrect solution to part a OR for reciprocals of each ratio</td>
</tr>
<tr>
<td>0</td>
<td>Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured</td>
</tr>
<tr>
<td>Blank</td>
<td>No response</td>
</tr>
</tbody>
</table>
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Score Point 2
(Example A)

6.

KLm = 160 cm  \[ \frac{16}{40} = \frac{2}{5} \]

\[ a \left( \frac{2}{5} \right) \]

\[ b. \frac{2^2}{5^2} = \frac{4}{25} \]

The student’s answer to each part is correct.

Score Point 2
(Example B)

6.

a) \[ \frac{2}{5} \]

b) \[ \frac{4}{25} \]

The student’s answer to each part is correct.

Score Point 2
(Example C)

6.

\[ \frac{16}{40} = \frac{4}{10} \]

\[ \frac{16^2}{40^2} \]

The student’s answer to each part is correct.
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**Score Point 1**

6. \[ A \leq \frac{3}{13.3} \]

The student's answer to part a is correct.
The student's answer to part b is incorrect.

6. \[ b \leq \frac{1}{2} \]

**Score Point 0**

6. \[ \frac{16}{40} = \frac{4}{10} = \frac{2^2}{5} = \frac{4}{25} \]

The student's answers to both parts are incorrect.

\[ \frac{16^3}{40^3} = \frac{8000}{64000} = \text{area} \]
Short Answer Response

11. A rectangle has a length of 5 feet and a width of 3 feet. When the length and width are each increased by the same amount, the perimeter is increased by 10 feet. By how much are the length and width increased?

Scoring Guide

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student gives correct answer, 2.5 (feet) or equivalent.</td>
</tr>
<tr>
<td>0</td>
<td>Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.</td>
</tr>
<tr>
<td>Blank</td>
<td>No response</td>
</tr>
</tbody>
</table>

Sample 1-Point Score

\[
11. \quad \begin{align*}
P &= 2(5) + 2(3) \\ P &= 16 \\
2(3+x) + 2(5+x) &= 16 + 10 \\
6 + 2x + 10 + 2x &= 26 \\
-16 &= -16 \\
x &= \frac{5}{2} \text{ ft.}
\end{align*}
\]

Sample 0-Point Score

11. It increased by the length so know it 10 so it increased by 5
12. Every 15 minutes on Thursday, Aisha counted the number of bank customers waiting in line. She made this bar graph from all of her observations.

![Bar Graph]

a. What was the median number of customers waiting in line on Thursday?

b. What was the mean number of customers waiting in line on Thursday?

**Scoring Guide**

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2 points</td>
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<tr>
<td>1</td>
<td>1 point</td>
</tr>
<tr>
<td>0</td>
<td>Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.</td>
</tr>
<tr>
<td>Blank</td>
<td>No response</td>
</tr>
</tbody>
</table>

**Training Notes:**

Part a: 1 point for the correct answer, 2

Part b: 1 point for the correct answer, 3.8
The median # of customers is 2. I know this because 2 is in the middle of the data:
0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 4, 4, 6, 8, 8, 9, 10, 10, 10.

The mean is all of the numbers averaged together.

The # of customers adds up to 95.

\[
\frac{95}{25} = 3.8 \rightarrow \text{The mean is about 4.}
\]
Sample 2-Point Score

12.

\[ \text{Median} = 2 \]
\[ \text{Mean} = 3.8 \]

Sample 1-Point Score

12.

The median number of customers was 4

\( \text{Total number of observations} = 25 \)
\( \text{Total number of customers} = 95 \)
\[ \frac{q_5}{25} = 3.8 \text{ customers} \]
Sample 1-Point Score

12.

A) 2 customers

B) 6 customers

Sample 0-Point Score

12.

a. \(2(0) + 5(1) + 6(2) + 3(3) + 2(4) + 1(6) + 2(8) + 1(9) + 3(10) = 87\)

\# of observations = 25

\(\frac{87}{25} = 3.48 = \text{median} = 3\)

b. mean = 2 because 2 was most common
13. A college recruiter compared the starting salaries for graduates with various majors. This table shows the results for two majors.

<table>
<thead>
<tr>
<th>Major</th>
<th>Mean Starting Salary</th>
<th>Median Starting Salary</th>
<th>Range of Starting Salaries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business</td>
<td>$39,000</td>
<td>$30,000</td>
<td>$20,000</td>
</tr>
<tr>
<td>Engineering</td>
<td>$35,000</td>
<td>$34,000</td>
<td>$12,000</td>
</tr>
</tbody>
</table>

Derrick will choose a major. He will decide between a business major and an engineering major. Use all of the information in the table to explain which major Derrick should choose.

Scoring Guide

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Student gives a complete argument based on the information in the table.</td>
</tr>
<tr>
<td>1</td>
<td>Student shows a minimal understanding of mean, median, and range.</td>
</tr>
<tr>
<td>0</td>
<td>Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.</td>
</tr>
<tr>
<td>Blank</td>
<td>No response</td>
</tr>
</tbody>
</table>

Sample Response:

Derrick should choose the engineering major, which has the higher median salary. Of the two, the business major has the higher mean starting salary; however, the range is significantly greater, which indicates that there might be an outlier. The median, therefore, would better represent the starting salaries of business majors. The mean and median starting salaries for engineering majors are about the same and the range is relatively small, which indicates a typical starting salary around $34,000–$35,000 vs. $30,000 for business majors.
Sample 2-Point Score

Derrick should choose to major in engineering. Though the mean or average starting salary for business is slightly higher, the median, or middle of the possible starting salaries is higher. Range indicates the difference between the greatest starting salary and the lower starting salary, and the range for engineering is quite a bit lower than the range for business. This indicates that the starting salaries for engineering are closer together, and a person who has an engineering major is more likely to get a higher starting salary than a business major.

Sample 1-Point Score

The business range is farther apart so the salary might be lower than the engineering. The median for engineering is higher so you know that the salary is better. Even though the mean salary is larger the median and the range give Derrick a better starting salary.

Sample 0-Point Score

He should choose engineering because the middle salary is lower but the average salary is higher and there is a smaller range of starting salaries.
16. Triangle $MNP$ has vertices at $M(0, 0)$, $N(12, 0)$, and $P(12, 16)$, as shown in the diagram below.

Let point $R$ be the midpoint of $MN$. Let point $S$ be the midpoint of $NP$.

a. What are the coordinates of each of the points listed below?
   - point $R$
   - point $S$

   Show or explain how you got your answer for each point.

b. Is $RS$ parallel to $MP$? Show or explain how you got your answer.

c. What is the length, in units, of each of the line segments listed below?
   - $MN$
   - $NP$
   - $MP$

   Show or explain how you got your answer for each line segment.

d. The length of $MP$ is how many times the length of $RS$? Show or explain how you got your answer.
16.

Scoring Guide and Sample Student Work
Select a score point in the table below to view the sample student response.

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4</strong></td>
<td>The student response demonstrates an exemplary understanding of the Geometry concepts involved in using rectangular coordinates to calculate midpoints and slopes of line segments, and distances between two points in the solution of a problem. The student determines whether or not two line segments are parallel, and calculates the ratio of the lengths of two line segments.</td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>The student response demonstrates a good understanding of the Geometry concepts involved in using rectangular coordinates to calculate midpoints and slopes of line segments, and distances between two points in the solution of a problem. Although there is significant evidence that the student was able to recognize and apply the concepts involved, some aspect of the response is flawed. As a result, the response merits 3 points.</td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>The student response demonstrates a fair understanding of the Geometry concepts involved in using rectangular coordinates to calculate midpoints and slopes of line segments, and distances between two points in the solution of a problem. While some aspects of the task are completed correctly, others are not. The mixed evidence provided by the student merits 2 points.</td>
</tr>
<tr>
<td><strong>1</strong></td>
<td>The student response demonstrates a minimal understanding of the Geometry concepts involved in using rectangular coordinates to calculate midpoints and slopes of line segments, and distances between two points in the solution of a problem.</td>
</tr>
<tr>
<td><strong>0</strong></td>
<td>The student response contains insufficient evidence of an understanding of the Geometry concepts involved in using rectangular coordinates to calculate midpoints and slopes of line segments, and distances between two points in the solution of a problem to merit any points.</td>
</tr>
</tbody>
</table>

Note: There are 2 sample student responses for Score Point 4.
16.

Sample 4-Point Score

a.) point \( R : (6,0) \)  

point \( S : (12,8) \)

I got this answer by drawing the triangle on the grid below, and then I used the grid to find the midpoints of MN and NP.

b.) Yes, \( \overline{RS} \) is parallel to \( \overline{MP} \). I used the grid below to find the slope, which is equal to \( \frac{rise}{run} \). I counted up 4 spaces and over 3 spaces which gives \( \overline{RS} \) and \( \overline{MP} \) a slope of \( \frac{4}{3} \).

c.) Length in units:

\[
\begin{align*}
\overline{MN} &= 12 \text{ units} \\
\overline{NP} &= 16 \text{ units} \\
\overline{MP} &= 20 \text{ units}
\end{align*}
\]

I got the lengths of \( \overline{MN} \) and \( \overline{NP} \) simply by referring to the numbers on the graph. To get the length of \( \overline{MP} \) I used the Pythagorean theorem, in which \( a^2 + b^2 = c^2 \).

\[
\begin{align*}
12^2 + 16^2 &= c^2 \\
144 + 256 &= 400 \\
20 &= c \\
\overline{MP} &= 20 \text{ units}
\end{align*}
\]

d.) \( \overline{MP} \) is twice the length of \( \overline{RS} \).

I knew the length of \( \overline{MP} = 20 \), so to get the length of \( \overline{RS} \) I used the Pythagorean theorem again.

\[
\begin{align*}
\overline{SN} &= 8 \text{ units}, \overline{EN} = 6 \text{ units}, \overline{ES} = c \\
\text{Pythagorean theorem: } a^2 + b^2 = c^2 \\
8^2 + 6^2 &= c^2 \\
100 &= 100 \\
\overline{RS} &= 10 \text{ units} \\
\overline{MP} &= 20 \text{ units}
\end{align*}
\]

\[
\frac{20}{10} = 2
\]

\( \overline{MP} \) is twice the length of \( \overline{RS} \).
### Sample 4-Point Score

2) The coordinate of point B is (6,0) - MN has a distance of 12 points on the graph, so to find the midpoint I divided 12 by 2 because the midpoint is in the middle of the line, the coordinate for point E is (12,8) because NP is 16 points on the graph, so point B is halfway between 0 and 16.

6) RS is parallel to MP.

The slope of MP is \( \frac{16-0}{12-6} = \frac{4}{6} = \frac{2}{3} \), and the slope from line RS is \( \frac{8-0}{12-6} = \frac{8}{6} = \frac{4}{3} \). The slopes are the same which means that the lines are parallel to each other.

C) The length of MN is 12 units, the length of NP is 16 units.

\[
\begin{align*}
\text{d} &= \sqrt{(0-0)^2 + (12-0)^2} \\
\text{d} &= \sqrt{144} \\
\text{d} &= 12
\end{align*}
\]

\[
\begin{align*}
\text{d} &= \sqrt{(16-0)^2 + (12-12)^2} \\
\text{d} &= \sqrt{256} \\
\text{d} &= 16
\end{align*}
\]

The length of MP is 20 units.

\[
\begin{align*}
\text{d} &= \sqrt{(16-0)^2 + (12-0)^2} \\
\text{d} &= \sqrt{256 + 144} \\
\text{d} &= \sqrt{400} \\
\text{d} &= 20
\end{align*}
\]

d) The length of MP is 2 times the length of RS.

\[
\begin{align*}
\text{d of RS} &= \sqrt{(8-0)^2 + (12-6)^2} \\
\text{d} &= \sqrt{64 + 36} \\
\text{d} &= \sqrt{100} \\
\text{d} &= 10
\end{align*}
\]

MP = 20 units RS = 10 units
16. Sample 3-Point Score

a) \( R = (4, 0) \) and \( S = (0, 8) \). I got \( R \) by dividing 12 by 2 and I got \( S \) by dividing 16 by 2.

b) Yes \( RS \) is parallel to \( MP \) because they have the same slope. The slope of \( RS \) is \( 4/3 \), and the slope of \( MP \) is \( 4/3 \).

c) \( MN \) is 12 units long. \( NP \) is 16 units long. \( MP \) is 20 units long.
\[
12^2 + 16^2 = (MP)^2 \quad 12^2 + 16^2 = 400 \quad \sqrt{400} = 20
\]
d) \( MP \) is twice as long as \( RS \).
\[
6^2 + 8^2 = 100 \quad \sqrt{100} = 10
\quad 20/10 = 2
16.

Sample 2-Point Score

a) The coordinate of point R is (6,0). The coordinate of point S is (12,8). I got point R by finding the midpoint of MN on the graph. I got point S by finding the midpoint of MP on the graph.

b) RS is parallel to MP. I got this by finding the slope of MP which was 8/4 and by finding the slope of RS which was also 8/4. If a slope is the same, the lines are parallel.

c) The length of MN is 12 units. The length of MP is 16 units and the length of RS is 18 units. I got those by counting the number of blocks it crosses on the grid.

d) The length of MP is 6 times the length of RS. I got my answer because I counted the units of the MR.

Point R is the midpoint of MN.
Sample 1-Point Score

16. Point R's coordinates would be (6, 0). I got this because (6, 0) is the coordinate directly in the middle of MN. Point S's coordinates would be (12, 9). I got this because (12, 9) is right in the middle of NP.

b) Yes, because when you draw a line connecting R and S it is parallel to NP.

c) The length of MN is 12 units. The length of NP is 16 units, and the length of MP is 16 units. I got these answers by looking at the last point on the diagram for example in MN I just looked on the diagram to see where point N was and it was at 12 units so line MN is 12 units long.

d) The length of RS is 9 units, and the length of MP is 16 units. And 16 - 9 = 7, so MP is 7 units longer than RS.
16.

Sample 0-Point Score

A) They just made the other triangle. I made the square for s (side).

B) Po, mp I added them together so it is 4 + 8 = 6

C) I cross them together and it is a rectangle

D) 4 times the size of the other shapes add them together