The Interactive Mathematics Program (IMP) is an alternative 4-year curriculum for high school mathematics. The IMP curriculum includes the same topics covered in the standard high school curriculum (Algebra I, Geometry, Algebra II, Pre-Calculus), but in an integrated format. It also includes topics in probability, statistics, discrete mathematics, and matrix algebra. The curriculum emphasizes the use of critical thinking, problem solving, communication, collaboration, and technology.

The IMP is closely aligned with the National Council of Teachers of Mathematics (NCTM) Principles and Standards documents.

COURSE DESCRIPTION:

The Interactive Mathematics Program (IMP) is a progressive four-year integrated, problem-centered mathematics curriculum. Graphing calculators and computer technology are used to enhance student understanding. The student explores open-ended situations in a way that closely resembles mathematics and scientists in their work.

In the fourth year of IMP, the student participates in a varied subject matter course of study rather than a calculus-focused one. Units build on the strong knowledge base of the student who has completed three years in the program. The student integrates concepts in linear projectile and circular motions using the graphing calculator, utilizes mathematics of computer animation, investigates families of function, develops and utilizes science of conducting polls, and extends and explores mathematical ideas. Problem settings include a Ferris® wheel circus act, election polling and programming an animated graphic display.

References in parentheses following each performance standard refer to and are aligned with the State Mathematics Standards (NM), Albuquerque Public Schools District Mathematics Standards (APS) located at www.aps.edu, and the APS Language Arts Standards.

Power standards are in italics.
STRATEGIES:
The “Illustrations” column in the Program of Studies provides exemplars of the performance standards, strategies, and best practices based on current educational research.

ASSESSMENTS:
The “Illustrations” column also incorporates a variety of assessments and “check for” items, suggested by APS mathematics teachers. Assessments include: authentic and performance-based assessment, cooperative learning, ongoing teacher observations, checklists, rubrics, formal and informal writing, self and peer assessments, small group and full class discussions, oral and multimedia presentations, projects, demonstrations, and portfolios/notebooks.

SUGGESTED TEXTBOOKS AND INSTRUCTIONAL MATERIALS:
- Graphing calculators
- *The Geometer's Sketchpad®*
- *Fathom Dynamic Statistics™*
- *Teaching Handbook for the Interactive Mathematics Program A Teacher-to-Teacher Guide* by Lori Green
- *IMPressions* newsletter is published by Key Curriculum Press each fall and spring for the Interactive Mathematics Program. You can request your free subscription online.

The following is available from Key Curriculum Press at http://www.keypress.com.

**IMP Year 3**
- *IMP Year 4 Student Textbook*
- *High Dive Teacher's Guide*
- *As the Cube Turns Teacher's Guide*
- *Know How Teacher's Guide*
- *The World of Functions Teacher's Guide*
- *Pollster’s Dilemma Teacher's Guide*
- *IMP Year 4 Calculator Guide for the TI-81, TI-82, and TI-83*

**Applicable to All IMP Years**
- Classroom Manipulatives Kit
- *It’s All Write: A Writing Supplement for High School Mathematics*

SUGGESTED WEBSITES:
- http://www.nctm.org - the National Council of Teachers of Mathematics site

Approved by HSCA: April, 2005
### GUIDING PRINCIPLES

The following Guiding Principles are extracted from the State of New Mexico Mathematics Curriculum Frameworks document adopted in June, 2002.

1. **Equity**: Excellence in mathematics requires equity, including high expectations and strong support for all students.
2. **Curriculum**: A curriculum is more than a set of activities—it must be coherent, focused on important mathematical content, and clearly articulated across grades.
3. **Teaching**: Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
4. **Learning**: Students must learn mathematics with understanding, actively acquiring new knowledge from experience and prior knowledge.
5. **Assessment**: Multiple and varied assessments should support the learning and furnish useful information to both teachers and students.
6. **Technology**: Technology is essential; it influences the mathematics that is taught and enhances student learning.

### ILLUSTRATIONS

**NOTE**: The student meets standards through creative classroom instruction, authentic assessments, high expectations, problem solving, multiple learning opportunities to apply skills, and technology support. The development and delivery of a successful mathematics program provides rich mathematical experiences accessible to ALL students.

The IMP curriculum is structured around learning activities that meet the standards identified in this document. These activities vary from year to year. The following activities are specifically designed for the IMP Year 4 curriculum and provide the basis for most of the illustrations used in the following strands.

**High Dive**

The central problem of this unit involves a circus act in which a diver is dropped from a turning Ferris wheel into a tub of water carried by a moving cart. The student’s task is to determine when the diver should be released from the Ferris wheel in order to land in the moving tub of water. In analyzing this problem, the student extends right-triangle trigonometric functions to the circular functions, studies the physics of falling objects (including separating the diver’s motion in its vertical and horizontal components), and develops an algebraic expression for the time of the diver’s fall in terms of his position. Along the way, the student identifies and then applies several additional trigonometric concepts, such as polar coordinates, inverse trigonometric functions, and Pythagorean identity.

**As The Cube Turns**

This unit opens with an overhead display generated by a program on a graphing calculator. The two-dimensional display depicts the rotation of a cube in three-dimensional space. The student learns how to write such a program, although the real focus is on the mathematics behind the program.
<table>
<thead>
<tr>
<th>GRADE</th>
<th>GUIDING PRINCIPLES</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The student studies the fundamental geometric transformations, – translations, rotations, and reflections – in both two and three dimensions, and expresses them in terms of coordinates. The study of these transformations also provides a new setting for the student to work with matrices, which he/she previously studied in connection with the systems of linear equations (in the Year 3 unit Meadows or Malls?). Another complex component of the student work is that he/she analyzes how to represent a three-dimensional object on a two-dimensional screen. As a concluding project, the student works in pairs to program an animated graphic display of his/her own design.

**Know How**

This unit is designed to prepare the student to find out independently about mathematical content he/she either has not learned or has forgotten. The student needs this skill in later education as well as in his/her adult work life. The student experiences the learning through reading traditional textbooks and interviewing other people. The student explores radian measure, ellipses, proof of the quadratic formula, the laws of sines and cosines, and complex numbers.

**The World Of Functions**

This unit builds on the student’s extensive previous work with functions. The student explores basic families of functions in terms of various ways they can be represented – as tables, as graphs, as algebraic expressions, and as models for real-world situations. The student also uses functions to explore a variety of problem situations and discovers that finding an appropriate function to use as a model sometimes involves recognizing a pattern in the data and other times requires insight into the situation itself. In the last portion of the unit the student explores ways of combining and transforming functions.

**The Pollster’s Dilemma**

The central limit theorem is the cornerstone of this unit in which the student looks at the process of sampling, with a special focus on how the size of the sample affects variation in pool results. The opening problem concerns an election poll which shows 53% of the voters favoring a particular candidate. The student investigates this question: How confident should the candidate be about her lead, based on this poll? By analyzing specific cases, the student sees that the results from a set of polls of a given size are approximately normally distributed. The student is given the statement of the central limit theorem which confirms this observation.
<table>
<thead>
<tr>
<th>GRADE</th>
<th>GUIDING PRINCIPLES</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
<td>Building on work in earlier units, the student learns how to use normal distributions and standard deviations to find confidence intervals and sees how concepts such as margin of error are used in reporting polling results. The student finishes the unit by working in pairs on a sampling project for a question of his/her own.</td>
</tr>
<tr>
<td>GRADE</td>
<td>PERFORMANCE STANDARDS</td>
<td>ILLUSTRATIONS</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>12</td>
<td><strong>The following State of New Mexico and APS Mathematics Standards align with the National Council of Teachers of Mathematics Standards (NCTM).</strong></td>
<td><strong>NOTE: Illustrations include suggested activities for attaining each performance standard. Key features to look for while assessing student performance are described in each unit.</strong></td>
</tr>
</tbody>
</table>

**Processes of Problem Solving**
1. Builds new mathematical knowledge through problem solving.
2. Solves problems that arise in mathematics and other contexts.
3. Applies and adapts a variety of appropriate strategies to solve problems (APS – I.2).
5. Draws on diverse knowledge and methods to solve problems (APS – I.3).

**Reasoning and Proof**
7. Recognizes reasoning and proof as fundamental aspects of mathematics.
10. Selects and uses various types of reasoning and methods of proof.

**Communication**
11. Organizes and consolidates their thinking through communication.
12. Communicates their mathematical thinking coherently and clearly to peers, teachers, and others (APS – I.9).
13. Analyzes and evaluates the mathematical thinking and strategies of others (APS – I.10).

<table>
<thead>
<tr>
<th>NOTE: Illustrations include suggested activities for attaining each performance standard. Key features to look for while assessing student performance are described in each unit.</th>
</tr>
</thead>
</table>
| 1 – 6. The student determines when a diver should be released from a moving Ferris wheel in order to land in the moving tub of water as described in the unit problem. (Unit problem from *High Dive*)
- problem-solving strategies
- analysis
- accuracy |

| 7 – 10. The student looks at the process of sampling, with a special focus on how the size of the sample affects variation in poll results. He/She responds orally or in writing to the question “How confident should a candidate be about his/her lead, based on a poll?” (Unit problem from *The Pollster’s Dilemma*)
- analysis/insights
- clear communication
- understanding of sampling |

| 11 – 14. The student responds to the following problem:
*Solution: Explain your results, including how you know that the number of moves for each number of discs is the smallest possible. Give any explanations you found for your generalizations.*
(Unit problem from *The Tower of Hanoi from High Dive*)
- clear and effective communication
- organization of thoughts/analysis |
<table>
<thead>
<tr>
<th>GRADE 12</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>14. Uses the language of mathematics to express mathematical ideas precisely.</td>
<td></td>
<td>√ appropriate mathematical language</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
<td>15. Recognizes and uses connections among mathematical ideas.</td>
<td>15 – 23. The student explores basic families of functions in terms of various ways they can be represented—as tables, as graphs, as algebraic expressions, and as models for real-world situations. The student also uses functions to explore a variety of problem situations and discovers that finding an appropriate function to use as a model sometimes involves recognizing a pattern in the data and other times requires insight into the situation itself. [Unit problem from <em>The World of Functions</em>]  √ connections  √ multiple representations  √ mathematical applications  √ problem-solving strategies</td>
</tr>
<tr>
<td>16. Understands how mathematical ideas interconnect and build on one another to produce a coherent whole.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19. Identifies how seemingly different mathematical situations may be essentially the same (e.g., the intersection of two lines is the same as the solution to a system of linear equations) (APS – I.13).</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Representation</strong></td>
<td>20. Creates and uses representations to organize, record, and communicate mathematical ideas.</td>
<td>24 – 26. Throughout the course, the student works in collaboration with fellow students, just as users of mathematics in the real world often work in teams. This process entails division of labor, discussion, and cooperation. [from <em>Note to Students</em>]  √ interaction with others  √ teamwork/cooperation  √ both individual and group participation</td>
</tr>
<tr>
<td>21. Selects, applies, and translates among mathematical representations to solve problems.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22. Uses representations to model and interpret physical, social, and mathematical phenomena.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23. Uses a variety of mathematical representations that can be used purposefully and appropriately interchangeably in all four years e.g., pictures, written symbols, oral language, real-world situations, and manipulative models) (APS – I.16).</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Collaboration</strong></td>
<td>24. Shares ideas and works cooperatively with others (APS – I.8).</td>
<td></td>
</tr>
<tr>
<td>25. Subdivides a task so that group members can work independently on different parts of it (APS – I.1).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26. Describes methods used to approach a problem (APS – I.4).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GRADE 12</td>
<td>PERFORMANCE STANDARDS</td>
<td>ILLUSTRATIONS</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------</td>
<td>----------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>√ effective communication</td>
</tr>
<tr>
<td></td>
<td></td>
<td>√ completion of tasks</td>
</tr>
<tr>
<td>GRADE 12</td>
<td>PERFORMANCE STANDARDS</td>
<td>ILLUSTRATIONS</td>
</tr>
<tr>
<td>----------</td>
<td>---------------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
|          | **Benchmark A:** The student represents and analyzes mathematical situations and structures using algebraic symbols. | 1. 13. The student finds at least five examples of even functions and five examples of odd functions making the examples for each category as varied as possible. He/She sketches the graphs of at least three functions in each category, explains what general properties the graphs of an even and odd function must have and gives an example of a function that is neither even nor odd.  
[Odd or Even from The World of Functions]
√ relevant examples
√ accurate graphs
√ correct properties for each type of function
√ effective communication |
[Absolutely Functions from The World of Functions]
√ research strategies
√ clear communication
√ understanding of absolute value |
|          | 2. Explains and uses the concept of absolute value (NM - I.A.10).                     | 3. The student simplifies a variety of rational expressions such as
\[ \frac{t^2 + 3t - 18}{t - 3} \]  
[A Fractional Situation from Know How]
√ accuracy
√ factoring techniques |
|          | 3. Simplifies fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms (NM - I.A.15). | 4. Using the given functions \( f(x) = x^2 + 3x + 4 \) and \( g(x) = 2x^2 - x + 1 \), the student defines the functions \( f + g, f - g, f \cdot g, \text{ and } f \div g \). |
|          | 4. Uses the four basic operations (+, -, x, ÷) with (NM - I.A.17):                   |                                                                                                                                           |
|          |   • polynomial expressions and                                                       |                                                                                                                                           |
## GRADE 12

### PERFORMANCE STANDARDS

- rational expressions.

**Benchmark B. The student understands patterns, relations, functions, and graphs.**

5. Determines whether a relation defined by a graph, a set of ordered pairs, a table of values, an equation, or a rule is a function (NM - I.B.2).

6. Analyzes and describes middle and end (asymptotic) behavior of linear, quadratic, and exponential functions, and sketches the graphs of functions (NM - I.B.10).

7. Works with composition of functions (e.g., find \( f \) of \( g \) when \( f(x) = 2x - 3 \) and \( g(x) = 3x - 2 \)), and finds the domain, range, intercepts, zeros, and local maxima or minima of the final function (NM - I.B.11).

### ILLUSTRATIONS

**[The Arithmetic of Functions from The World of Functions]**
- √ applications of basic operations
- √ accuracy

5. The student selects a unit previously studied and a specific function from that unit and (1) describes the problem context in which the function was used and explains what the input and output for the function represent in terms of the problem context; (2) describes how the function was helpful in solving the central unit problem or some other problem in the unit; and (3) tells, if possible, from which family the function is.

**[What Good Are Functions? From The World of Functions]**
- √ understanding of functions
- √ all required components
- √ accuracy
- √ effective communication

6, 9, 10. The student looks at polynomials of different degrees and, in each case, determines what happens to the \( y \)-value as \( x \) increases in absolute value. Next, he/she looks at functions from other families: exponential functions, functions from the sine family, rational functions, and any other families he/she wants to consider and makes the same determination.

**[The End of the Function from the World of Functions]**
- √ investigations
- √ conjectures
- √ accuracy
- √ connections

7. The student responds to the following scenario:

There are two functions involved in this situation. One function gives \( c \), the number of catfish per acre, as a function of \( w \), the number of gallons of toxic waste dumped per day. For this activity, suppose that \( c = 50 - 5w \) and write this function as \( c = G(w) \), so \( G \) is defined by the formula \( G(w) = 50 - 5w \). The second function expresses \( p \), the price (in dollars) of a catfish dinner, as a function of \( c \), the number of catfish per acre. For this activity, suppose that \( p = 24 - 0.4c \) and write this function as \( p = H(c) \), so \( H \) is defined by the formula \( H(c) = 24 - 0.4c \). Write an equation expressing \( p \) in terms of \( w \), and represent this equation symbolically using composition notation.

**[The Cost of Pollution from The World of Functions]**
<table>
<thead>
<tr>
<th>GRADE 12</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
</table>
| Benchmark C: The student uses mathematical models to represent and understand quantitative relationships. | 8. Models real-world phenomena using linear and quadratic equations and linear inequalities (e.g., apply algebraic techniques to solve rate problems, work problems, and percent mixture problems; solves problems that involve discounts, markups, commissions, and profit and compute simple and compound interest; applies quadratic equations to model throwing a baseball in the air) (NM - I.C.1). | √ correct functional notation  
√ understanding of composition of functions |
| 9. Uses a variety of computational methods (e.g., mental arithmetic, paper and pencil, technological tools) (NM - I.C.2). | 10. Evaluates numerical and algebraic absolute value expressions (NM - I.C.6). | 8, 11, 12. The student solves the following problem: For a science experiment in biology, Binh is observing the growth of bacterial colony. At 2 p.m., he estimates that there are 1000 bacteria. When he returns to check at 5 p.m., there are about 2200. If the bacteria continue to reproduce at this rate, how many will there be at midnight? Find an exponential function that will tell you how many bacteria will be present at a given time. |
| 11. Generates an algebraic sentence to model real-life situations (NM - I.C.9). | Benchmark D: Analyzes change in various contexts. | [Families Have Many Different Members from The World of Functions] |
| 12. Solves routine two- and three-step problems relating to change using concepts such as (NM - I.D.2): Exponents | 13. Analyzes the general shape of polynomial expressions and equations for different degree polynomials (e.g., positive and negative general shapes for third-, fourth-, and fifth-degree polynomials) (NM - I.D.4). | √ relevance  
√ connections  
√ problem-solving strategies  
√ accuracy |
| 14. Evaluates the estimated rate of change in the context of the problem (NM - I.D.6). | 14. The student solves a variety of problems similar to the following: A driver averaged 60 miles per hour on her 100-mile trip to the beach. What should her average speed be on the return trip so that her average speed for the round trip will be 50 miles per hour? (Warning: The answer is not 40 miles per hour!) [Homework 11: An Average Drive from The World of Functions] | √ accuracy  
√ understanding of rate of change |
| 15. Uses regression to find the function that best fits a set of data (NM – IIIB). | 15. The student solves the following problem: | |
16. Finds the inverse of a function (NM – IV - Guidance for further study).

17. Applies transformations of functions to graphs (NM – IIC.1, 2).

---

**ADDITIONAL TOPICS**

**Solving Equations, Inequalities and Systems**

(As the student encounters ever more sophisticated mathematical situations, he/she needs to be able to generate and solve a variety of equations.)

---

Steve was looking at the relationship between the length of certain models and the amount of paint they require. Here are some of the estimates he came up with.

<table>
<thead>
<tr>
<th>Length of model (in feet)</th>
<th>Amount of paint (in ounces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
</tr>
</tbody>
</table>

Assuming that this pattern continues, estimate how much paint would be needed for a model that was 10 feet long. Explain.

[Let’s Regress from The World of Functions]

- √ accuracy
- √ effective communication

16. The student finds the inverse (under composition) for the function \( k \) defined by the equation \( k(x) = 4x^3 – 5 \) and shows how the answer is the inverse.

[Homework 26: An Inventory of Inverses from The World of Functions]

- √ accuracy
- √ documentation of work

17. The student describes how the graphs of each of the following transformations of \( y = f(x) \) differ from the graph of \( y = f(x) \).

- \( y = f(x) – 3 \)
- \( y = f(x + 2) \)
- \( y = 1/2 f(x) \)
- \( y = f(2x) \)

[Homework 28: Transforming Graphs, Tables, and Situations from The World of Functions]

- √ understanding of transformations
- √ effective communication

**Illustrations for Additional Topics**

See Strand I – Guiding Principles, the description for The World of Functions.
inequalities, and systems. He/She begins by studying more complex linear and quadratic equations and systems.) The student (NM - IV.1):
- solves quadratic inequalities by factoring.

Polynomials
(The student extends the concept of solving linear equations to higher degree polynomials. These polynomials can be used to more accurately describe real-world phenomena.) The student (NM - IV.2):
- factors polynomials of degree higher than two using the fundamental theorem of algebra (e.g., an nth degree polynomial has at most n distinct linear factors), integral and rational zero theorems, and factor and remainder theorems,
- performs the four basic operations on complex numbers,
- factors polynomials using complex numbers,
- graphs polynomials using the intermediate value theorem, and
- graphs and interprets the conic sections.

Functions
(The language and properties of functions are essential to understanding the components of higher mathematics. Functions are the fundamental objects on which students operate in some higher mathematics and are among the building blocks of higher mathematics.) The student (NM - IV.3):
- finds and uses inverse functions involving ordered pairs, graphs, and explicit statements of a function rule and
- examines and graphs piece-wise defined functions, including the use of the properties of continuity and discontinuity.

Logarithms and Exponential Functions
(Logs and exponential functions provide tools for more sophisticated modeling and applications for understanding real-life phenomena. Higher mathematics requires regular and successful use of logs and exponents to move beyond polynomials.) The student (NM - V.1):
- operates with logs and exponential functions on the basis of their inverse relationship
- identifies the concept of \( e \),
- uses exponential functions and common and natural logs to understand real-life situations (e.g., half-life, amortization, logistic growth), and
- uses logs and exponents to solve equations.
<table>
<thead>
<tr>
<th>GRADE</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Series and sequences</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[As the student progresses toward higher mathematics, he/she needs an understanding of sequences functions whose domains are sets of whole numbers as opposed to sets of real numbers (e.g., discrete functions versus continuous functions). Infinite geometric series provide one way to begin a discussion about limits.) The student (NM - V.3):</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• uses algebraic techniques to generate the specific formulas for arithmetic and geometric sequences and series,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• extends the concept of series to infinite geometric series,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• uses the language and notation of limits, and</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• uses mathematical induction to prove various mathematical statements.</td>
<td></td>
</tr>
</tbody>
</table>
**STRAND IV: GEOMETRY AND TRIGONOMETRY**

**CONTENT STANDARD:** The student understands geometric concepts and applications.

**BENCHMARKS:**

A. The student analyzes characteristics and properties of two- and three-dimensional geometric shapes and develops mathematical arguments about geometric relationships.

B. The student applies transformations and uses symmetry to analyze mathematical situations.

<table>
<thead>
<tr>
<th>GRADE 12</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
</table>
|          | **Benchmark A:** Analyzes characteristics and properties of two- and three-dimensional geometric shapes and develops mathematical arguments about geometric relationships.  
1. Interprets and draws three-dimensional objects and finds the surface area and volume of basic figures (e.g., spheres, rectangular solids, prisms, polygonal cones), and calculates the surface areas and volumes of these figures as well as figures constructed from unions of rectangular solids and prisms with faces in common, given the formulas for these figures (NM - IIA.4). | 1. The student performs the following steps for programming his/her calculator to rotate a cube on the screen:  
- Draw a picture on the graphing calculator.  
- Create the appearance of motion.  
- Change the position of an object located in a two-dimensional coordinate system.  
- Create a two-dimensional drawing of a three-dimensional object.  
- Change the position of an object located in a three-dimensional coordinate system.  
[An Animated Shape from As the Cube Turns]  
√ ability to follow directions  
√ desired outcome  
√ individual participation  
√ use of technology |
|          | **Benchmark B:** Applies transformations and uses symmetry to analyze mathematical situations.  
2. Describes the effect of rigid motions on figures in the coordinate plane and space that include rotations, translations, and reflections (NM - IIC.1):  
- determines whether a given pair of figures on a coordinate plane represents the effect of a translation, reflection, rotation, and/or dilation and  
- sketches the planar figure that is the result of a given transformation of this type.  
3. Deduces properties of figures using transformations that include translations, rotations, reflections, and dilations in a coordinate system (NM - IIC.2):  
- identifies congruency and similarity in terms of transformations | 2, 3, 5. The student draws a triangle, reflects the triangle about the line \( y = x \), generalizes finding the image of an arbitrary point \((a, b)\) under the reflection through the line \( y = x \), and expresses the reflection of the triangle in terms of matrices. That is, he/she finds a matrix process that turns the vector \([a, b]\) into the corresponding image vector under the reflection. [Homework 26: Further Flips from As the Cube Turns]  
√ all required components  
√ individual participation  
√ accuracy  
√ understanding of transformations |
<table>
<thead>
<tr>
<th>GRADE 12</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>and</td>
<td>• determines the effects of the above transformations on linear and area measurements of the original planar figure.</td>
<td>4. The student explains in his/her own words what a radian is and states some questions he/she has about radians. [Radian Measure from Know How] ✓ understanding of radians ✓ effective communication</td>
</tr>
<tr>
<td>5. Applies the laws of sines and cosines (NM – V – Guidance for further study).</td>
<td>6. The student uses the law of sines and the given information to solve triangle ABC rounding off angles and lengths to the nearest tenth. ( a = 30 \text{ cm}, \beta = 50^\circ, \gamma = 35^\circ ) [Homework 2: The Law of Sines from Know How] ✓ application of law of sines ✓ accuracy (e.g., precision, solution)</td>
<td></td>
</tr>
<tr>
<td>6. Applies the laws of sines and cosines (NM – V – Guidance for further study).</td>
<td>7. The student chooses two values for ( \theta ), in different quadrants, that have the same value for ( \cos \theta ). [Homework 14: Oh, Say What You Can See from As the Cube Turns] ✓ accuracy ✓ problem-solving strategies</td>
<td></td>
</tr>
<tr>
<td>7. Uses trigonometric identities to solve problems involving angles (NM – V – Guidance for further study).</td>
<td>8. Derives formulas for the sine and cosine of sums and differences of angles (NM – V – Guidance for further study).</td>
<td>8. Given the following equation of an ellipse, ( 16x^2 + 25y^2 ), the student finds the coordinates of the vertices and critical points of the ellipse, the lengths of the major and minor axes, the coordinates of the foci, and sketches the ellipse. [Homework 3: The Ellipse from Know How] ✓ all required components ✓ accuracy ✓ reasonable sketch ✓ understanding of ellipses</td>
</tr>
<tr>
<td>8. Relates the equation of an ellipse to the axes, foci and critical points of an ellipse (NM – IV - Guidance for further study).</td>
<td>9. The student develops a formula for ( \sin(A + B) ), where ( A ) and ( B ) are two angles, using ( \sin A, \cos A, \sin B, ) and ( \cos B ) in the formula. [The Sine of a Sum from As the Cube Turns] ✓ reasoning ✓ correct formula</td>
<td></td>
</tr>
</tbody>
</table>
STRAND V: DATA ANALYSIS AND PROBABILITY
CONTENT STANDARD: The student understands how to formulate questions, analyze data, and determine probabilities.

BENCHMARKS: A. The student formulates questions that can be addressed with data and collects, organizes, and displays relevant data to answer them.
B. The student selects and uses appropriate statistical methods to analyze data.
C. The student develops and evaluates inferences and predictions that are based on data.

<table>
<thead>
<tr>
<th>GRADE</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
</table>
| 12    | **Benchmark A:** The student formulates questions that can be addressed with data and collects, organizes, and displays relevant data to answer them.  
1. Understands the differences between the various methods of data collection (NM - III.A.1).  
2. Knows the characteristics of a well-designed and well-conducted survey (NM - III.A.2):  
   - differentiates between sampling and census and  
   - differentiates between a biased and an unbiased sample.  
3. Knows the characteristics of a well-designed and well-conducted experiment (NM - III.A.3):  
   - differentiates between an experiment and an observational study and  
   - recognizes sources of bias in poorly designed experiments.  
4. Understands the role of randomization in well-designed surveys and experiments (NM - III.A.4). | 1 – 4. The student examines how well polls of different sizes reflect the reality of a population by simulating a poll. He/She uses 150 objects to represent 150 students. The objects should be identical except for color.  
90 of the objects (60% of the population) are of one color and the remaining objects a different color. The student chooses a sample size. Without looking into the bag, he/she picks objects from the bag and records the number of votes for the 60% population. When done, he/she returns all of the objects to the bag and repeats this sampling process for 20 polls of the same sample size. The student makes a frequency bar graph of the results of the 20 polls and determines how many of the polls favored the 60% group. As time allows, the student repeats the above procedure for other sample sizes. Based on the results, the student determines how small a group can be polled and still get a good idea of what the majority prefer. [Sampling Seniors from the Pollster’s Dilemma]  
   √ conclusion  
   √ experimentation  
   √ adherence to procedures |
|       | **Benchmark B:** The student selects and uses appropriate statistical methods to analyze data.  
5. Calculates and interprets the mean and standard deviation for a probability distribution (NM – IIIB.3). | 5. The student finds the mean and standard deviation for the three-coin probability distribution. [Homework 11: A Distribution Example from The Pollster’s Dilemma]  
   √ calculations  
   √ accuracy |
|       | **Benchmark C:** The student develops and evaluates inferences and predictions that are based on data.  
6. Draws conclusions concerning the relationships among bivariate data (NM III.C.2): | 6 - 8. The student solves the following problem:  
Suppose you take a 20-person poll \( (n = 20) \) and get a sample proportion of |
<table>
<thead>
<tr>
<th>GRADE 12</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
</table>
| **•** determines the strength of the relationship between two sets of data by examining the correlation and understands that correlation does not imply a cause-and-effect relationship. |  | \( \hat{p} = 0.60 \). Suppose the true proportion \( p \) is 0.55. What is the value of the standard deviation \( \sigma \)? What are the endpoints of the 95% confidence interval around \( \hat{p} \)? Is the true proportion \( p \) within the 95% confidence interval?  
*Different \( p \), Different \( \sigma \) from The Pollster’s Dilemma* |
<p>| 7. Uses simulations to explore the variability of sample statistics from a known population and construct sampling distributions (NM - III.C.3). |  | √ response to questions |
| 8. Understands how sample statistics reflect the values of population parameters and uses sampling distributions as the basis for informal inference (NM - III.C.4). |  | √ reasoning |
| 9. Evaluates published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions (NM - III.C.5). |  | √ accuracy |
| 10. Applies the central limit theorem to the size of polling samples (NM – V – Guidance for further study). |  |  |
| 11. Plots and interprets graphs of normal curves of the type |  |  |
| [ y = \left( \frac{1}{\sigma \sqrt{2\pi}} \right) e^{-\frac{(x-\mu)^2}{2\sigma^2}} ] (NM – V - Guidance for further study). |  |  |</p>
<table>
<thead>
<tr>
<th>GRADE 12</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>different means and records changes seen.</td>
<td>[Graphing Distributions from <em>The Pollster’s Dilemma</em>]</td>
</tr>
<tr>
<td></td>
<td>√ use of technology</td>
<td></td>
</tr>
<tr>
<td></td>
<td>√ recordkeeping/observations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>√ graphical representations</td>
<td></td>
</tr>
</tbody>
</table>
**STRAND VI: LITERACY**

**CONTENT STANDARD:** The student communicates mathematical principles through reading, writing, speaking, and research opportunities.

**BENCHMARK:** The student demonstrates through a variety of writing and speaking requirements proficiency in reading comprehension, specialized vocabulary, and reasoning.

<table>
<thead>
<tr>
<th>GRADE 12</th>
<th>PERFORMANCE STANDARDS</th>
<th>ILLUSTRATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The student focuses on 12th grade language arts standards unless otherwise indicated.</td>
<td>Note: The very nature of IMP courses, as the name suggests, requires the student to interact; to engage in activities that enhance his/her vocabulary of mathematics; to communicate symbolically, orally, and in written formats; and to think critically through problem-solving situations. Through consistent integration of the mathematical processes, the student works collaboratively with other students, requiring whole or small group discussions; listens to other’s viewpoints whether it be via print, technology, or guest speaker; displays data in an organized fashion; and makes connections. Consequently, literacy strategies are integrated and reflected in every strand. The following citations illustrate specific examples of these strategies although numerous opportunities are presented throughout the year and throughout the curriculum.</td>
</tr>
<tr>
<td>1.</td>
<td>Demonstrates command of reading strategies across content areas (APS – LA I.1).</td>
<td>1, 2. See each strand and its numerous word problems and Strand V, the illustration for performance standard #8.</td>
</tr>
<tr>
<td>2.</td>
<td>Evaluates the author’s use of argument to support intended purpose (APS – LA II.8).</td>
<td>3, 4. See Strand II, 2nd illustration; Strand III, the illustration for performance standards #6, #9, #10; Strand IV, the 1st illustration; and Strand V, the illustration for performance standard #10.</td>
</tr>
<tr>
<td>3.</td>
<td>Demonstrates fluency in using the writing process to create a final product (APS – LA III.1).</td>
<td>5 – 7. See Strand II, the 2nd and 3rd illustrations; Strand III, the 1st, 2nd and 5th illustrations and the illustrations for performance standards #15 and #17; Strand IV, the illustration for performance standard #4; and Strand V, the illustration for performance standard #8.</td>
</tr>
<tr>
<td>4.</td>
<td>Demonstrates fluency in using a variety of technology (APS – LA III.3).</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Demonstrates fluency with speaking strategies (APS – LA IV.1).</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Participates in group discussions and/or presentations to the class (APS – LA V.2 – Gr. 9).</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Applies criteria to evaluate oral presentations and arguments (APS – LA IV.2).</td>
<td></td>
</tr>
<tr>
<td>GRADE 12</td>
<td>PERFORMANCE STANDARDS</td>
<td>ILLUSTRATIONS</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------</td>
<td>---------------</td>
</tr>
<tr>
<td>8.</td>
<td>Selects information to incorporate, use, and convey ideas in new ways (APS – LA V.1).</td>
<td>8, 9. See Strand II, the illustration for performance standards #15 – #23; Strand IV, the 2nd illustration; and Strand V, the 1st illustration and the illustration for performance standard #8.</td>
</tr>
<tr>
<td>9.</td>
<td>Uses a variety of sources to gather information and synthesize ideas (APS – LA VI.1).</td>
<td></td>
</tr>
</tbody>
</table>