PARCC MODEL CONTENT FRAMEWORKS

MATHEMATICS

GRADES K–2

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Introduction to the PARCC Model Content Frameworks for Mathematics: Kindergarten through Grade 2

The Common Core and Early Childhood Education

States have had standards for K–2 mathematics for a long time—since well before the development of the Common Core State Standards. The Common Core State Standards build on states' longstanding work in the primary grades and are designed to provide young students with a rich, rewarding, rigorous, and equitable education.

Experts in early childhood mathematics education were directly involved in developing the Common Core State Standards for Mathematics. The standards were also informed by a range of evidence, including peer-reviewed empirical studies, summaries of research in education, recommendations from mathematicians and mathematics educators, expectations from previous state standards documents, expectations from high performing countries, and domestic reports such as *Adding It Up* and *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*. Drafts of the standards were reviewed by experts in early childhood education as well as by current and former elementary-grades teachers.

The Standards are not a checklist of separate to-dos. For example, in Kindergarten, a single learning experience involving numbers might help students make progress toward several standards at once (e.g., K.CC.B.4, K.CC.C.6, and K.OA.A.2).

The content standards are *end-of-grade expectations*. Moreover, the order of the content standards doesn't dictate an order in which the indicated material is to be taught. For example, content standard K.CC.A.1 sets an expectation that Kindergarten students will count to 100 by ones and by tens. This standard is the first one listed, but it is by no means the first standard a student would typically meet during the Kindergarten year. This is a Kindergarten example, but the point is relevant in every grade as well as high school.

Standards and curriculum are two different things. The curriculum should enable all students to meet the standards in a supportive environment that emphasizes grade-appropriate understanding with explaining of concepts and fluency with key grade-level mathematical operations like counting, addition, and subtraction.

Focus in Grades K–2.[1]

For years, national reports have called for greater focus in U.S. mathematics education. TIMSS and other international studies have concluded that mathematics education in the United States is a mile wide and an inch deep. A mile-wide inch-deep curriculum translates to less time per topic. Less time means less depth and moving on without many students.

The strong focus of the Standards in early grades is arithmetic along with the components of measurement that embody it. That includes the concepts underlying arithmetic, the skills of arithmetic computation, and the ability to apply arithmetic to solve problems and put arithmetic to engaging uses. Arithmetic in the K–5 standards is an important life skill, as well as a thinking subject and a rehearsal for algebra in the middle grades.

By design, some topics familiar to primary grades teachers are absent from the K–2 standards. Implementing the standards requires moving some topics traditionally taught in earlier grades up to higher grades—sometimes to much higher grades. "Teaching less, learning more" can seem counterintuitive. But remember that the goal of focus is to make good on the ambitious promise the states have made to their students by adopting the Standards: greater achievement at the college- and career-ready level, greater depth of understanding of mathematics, and a rich classroom environment in which reasoning, sense-making, applications, and a range of mathematical practices all thrive. None of this is realistic in a mile-wide, inch-deep world

Focus might not matter so much if arithmetic were nothing more than a list of procedures to memorize. But even very early work with numbers is rich and intricate. For example, there is much more to counting than initially meets the eye. It takes time for a student to transition from rote counting (merely reciting the number words) to cardinal counting (telling how many). Until this transition happens, students might not reliably pair numbers 1-1 with objects (K.CC.B.4a) and might not realize that the last number said tells the number of objects (K.CC.B.4b). Another aspect of cardinal counting is to understand that each successive number name refers to a quantity that is one larger (K.CC.B.4c). For example, if a student knows there are 5 marbles in her hand, and another marble is placed in her hand, does she find the new total by counting all of the marbles again starting from 1, or does she simply say "6"? (Note that the latter cannot happen unless a student also knows how to rote-count forward from a number other than 1 (K.CC.A.2).) Finally, cardinal counting is a mental-physical performance that takes practice. Students can be helped to use techniques that lead to more accurate counting, such as counting objects from left to right, setting aside the already counted objects, saying "first I'll do the ducks, then the geese," and so on.

In arithmetic, there are intricate, difficult, and necessary things that also form prerequisites for intricate, difficult, and necessary things. Think about such aspects of mathematics as the linguistic patterns (and the deviations from those patterns) in the count sequence; the place value system, with its three linked notions of base ten units, recursive bundling and unbundling, and positional notation; and the 15 basic kinds of addition and subtraction situations (and that is just the one-step problems). Each of these things is a world in itself, and additionally the connections between these areas are dense.

Without the foundations of number in place, students are unlikely to succeed in later mathematics. Research shows that early number sense predicts later functional literacy, which in turn matters for students' future life outcomes. Research also shows how early number sense predicts later math scores for all content strands. Focusing on the major work of each grade, as detailed in this Model Content Framework, helps keep students on track to college and career readiness. Therefore to give young children the time they need to learn these fundamentals thoroughly in a grade-appropriate environment, the primary grades focus strongly on the major work of the grade, detailed further below in this Model Content Framework.

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[1] Parts of this section are excerpted and adapted from the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics, http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

Kindergarten Preface

Students with academically oriented families and/or preschool experience may enter kindergarten able to recite number words and use *cardinal counting*, that is, counting to answer "how many?" questions. Kindergarten establishes these and other foundations and begins the process of building the mathematical habits of mind that lead to proficient mathematical practice and are described in the Standards for Mathematical Practice.

Early number work is rich and intricate. Cardinality and other essential ideas of kindergarten take time to develop. For example, the transition from rote counting to cardinal counting takes time; until this transition happens, students might not reliably pair numbers 1-1 with objects (K.CC.B.4a) and might not realize that the last number said tells the number of objects (K.CC.B.4b). Another aspect of cardinal counting is to understand that each successive number name refers to a quantity that is one larger (K.CC.B.4c). For example, if a student knows there are 5 marbles in her hand, and another marble is placed in her hand, does she find the new total by counting all of the marbles again starting from 1, or does she simply say "6"? Cardinal counting is also a mental-physical performance that takes practice. Students can be helped to use techniques that lead to more accurate counting, such as counting objects from left to right, setting aside already counted objects, saying "first I'll do the ducks, then the geese," etc. Kindergarten students also learn to handle situations of comparing, adding and subtracting.

Without these foundations in place, students are unlikely to succeed in later mathematics.[2] To give young children the time they need to learn these fundamentals in an age-appropriate environment of play and exploration, kindergarten focuses strongly on number; see "Content Emphases by Cluster" below.[3]

Examples of Key Advances in Kindergarten

- Students learn to pair objects 1-1 with counting words, and they learn that the last number word tells the number of objects in a collection (up to 20). This is called "cardinal counting," as opposed to "rote counting" (reciting the counting words in order).
- Students will use their ability to subitize (recognize small quantities at a glance) to help them compose and decompose numbers. For example, when using objects to show the decompositions 5 = 2 + 3 or 12 = 10 + 2, it is helpful for students to subitize two or three objects.
- Some students will progress from the "counting all" strategy to the more sophisticated strategy of "counting on" during kindergarten (see K.CC.B.4c and 1.OA.C.5).

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[2] See Geary et al. 2013. "Adolescents' Functional Numeracy Is Predicted by Their School Entry Number System Knowledge." PLoS ONE 8(1) and Jordan et al. 2009. "Early Math Matters: Kindergarten Number Competence and Later Mathematical Outcomes." *Developmental Psychology* 45(3):850–867.

[3] For more information, see the *Progression* document for K Counting and Cardinality and K–5 Operations and Algebraic Thinking. Available at www.ime.arizona.edu/progressions.

- Students learn to compare the number of objects in one group versus another group and eventually to compare written numerals 1–10.
- Students understand addition as joining collections and adding to collections, and they understand subtraction as taking collections apart or taking from collections, representing these operations in a variety of ways.

Fluency Expectations or Examples of Culminating Standards

K.CC.A.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

It is recommended that students work throughout the year toward fluency in writing the numerals 0–10. Note that learning to write numerals is generally more difficult than learning to read them. The easier numerals 1, 3, 4, 5 and 7 can often be mastered earlier. It is common for students to reverse numbers at this stage (e.g., writing Ɛ for 3).[4]

K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

It is recommended that students become fluent in cardinal counting well before the end of kindergarten because much else in kindergarten depends on it.

K.CC.C.7 Compare two numbers between 1 and 10 presented as written numerals.

If students are less than fluent in number comparisons by the end of kindergarten, then they may not have mastered early number concepts. Note that standard K.CC.C.6 is a precursor to K.CC.C.7 and portrays the kind of concrete work students should be doing en route to mastering numeral-based comparisons.

K.OA.A.5 Fluently add and subtract within 5.[5]

That is, given any two numbers 0–5 with a sum less than or equal to 5, students can name the sum reasonably quickly; and likewise for related differences, given one number and a goal that is 5 or less, they can reasonably quickly name the "missing amount." Students grow in fluency throughout the year as they work with addition and subtraction situations.

Examples of Major Within-Grade Dependencies

- Much of the learning in kindergarten — K.CC.C.6, all of K.OA and K.NBT, and K.MD.B.3 — depends on the foundational ability to count to answer "how many?" (K.CC.B.5), which itself is grounded in K.CC.B.4.

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[4] Material adapted from *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council (2009), page 138.

[5] Wherever the word *fluently* appears in a content standard, the word means quickly and accurately.

Examples of Opportunities for Connections among Standards, Clusters or Domains

- In addition to laying the groundwork for place value in grade 1, working with numbers 11–19 (K.NBT) provides opportunities for cardinal counting beyond 10 (see K.CC.B.5) and for writing two-digit numbers (see K.CC.A.3). Ten frames can be helpful for this work.
- K.MD.B.3 provides opportunities for cardinal counting (see K.CC.B.5) and for comparing numbers (see K.CC.C.6). K.MD.B.3 also offers a context in which to decompose 10 in more than one way (see K.OA.3).
- K.G.A.2 and K.G.B.4 offer some opportunities for counting and comparing numbers.

Examples of Opportunities for In-Depth Focus

K.CC.B.5 Cardinal counting is a focus in itself as needed and is a main component of other work in the kindergarten classroom. Opportunities to develop students' understanding of cardinality abound, both within the instructional time devoted specifically to mathematics and elsewhere in the instructional day. For example, how many plants did the class plant for the science project? What if we planted one more?

K.OA.A.2 Through representing and solving addition and subtraction problems, students understand addition as joining and adding to and understand subtraction as separating and taking from. Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time, with subtraction coming to be understood as reversing the actions involved in addition and as finding an unknown addend.[6]

K.OA.A.3 Connected with other standards such as K.OA.A.1 and K.OA.A.2, the ability to decompose numbers flexibly is a key focus. At this age, children who seem competent at counting can hold conversations like this:

— Happy birthday, Naomi! Tell the class how old you are today.

— I'm *six!* (Naomi shows .)

— Oh, so you are *this* old? (Teacher shows .)

— (Naomi giggles.) *No!* I'm *this!* (And she reshows .)

— Oh, so are you *this* old, right? (Teacher shows .)

— (Naomi giggles again.) *Noooo!* I'm *this old!* (And she again shows .)

Naomi seemed to believe there was only one correct way to show 6.

[6] See pages 2 and 3 of the *Progression* document for K Counting and Cardinality and K–5 Operations and Algebraic Thinking. Available at www.ime.arizona.edu/progressions.

K.OA.A.4 **"**Making ten" will become a key strategy (in grade 1) for adding and subtracting within 20; students gain the foundations for this in kindergarten by finding the number that makes 10 when given another number. Over the course of the year, given frequent opportunities (e.g., a "how many fingers *don't* you see" game), many kindergarten children can become fluent with the pairs of numbers that make 10 and can, when a number less than 10 is named, name the "missing amount" even without looking at fingers.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practice should be evident throughout mathematics instruction and connected especially to the most important work of the grade. Mathematical tasks (short, long, scaffolded and unscaffolded) are an important opportunity to connect content and practices, as are class discussions. Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

- Kindergarten students *speak* the number names by ones and by tens all the way to 100 (K.CC.A.1). The structure of a number name like "thirty-two" reflects the underlying system of place value. Attending to and using that structure (MP.7) is, even more than K.NBT.A.1, an important foundation for place value. For example, counting by 10s not starting at 0 can let students hear the patterns in the number names (e.g., "thirty-eight, forty-eight, fifty-eight, sixty-eight …"). Hearing this structure prepares them to read those numbers in grade 1. See the *Progression* document for K Counting and Cardinality and K–5 Operations and Algebraic Thinking for more information about how patterns in the number names affect learning (including deviations from those patterns, as in "sixteen," which puts the ones digit first).

- As children learn to count by tens (K.CC.A.1), they may make sense (MP.1) of these numbers by reciting each new number in the sequence ten, twenty, thirty … as a new child joins the ones already standing in front of the classroom and showing all their fingers. The patterns in the place value system — the structure of numbers (MP.7) — becomes apparent when students connect *six* children with the spoken *sixty*, *seven* children with the spoken *seventy* and *eight* children with the spoken *eighty*.

- When students progress from drawing realistic (artistic) pictures of situations to diagramming addition and subtraction situations using circles or other symbols, they are relating the concrete to the abstract (MP.2) and making their first mathematical models (MP.4). The equations that the teacher writes on the board to describe these situations (such as 8 + 2 = 10) are also mathematical models.

- If a student chooses to use objects, fingers or a math drawing to analyze and solve a word problem, then it is an example of the student using an appropriate tool strategically (MP.5).

A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens and ones are an important part of the K–2 classroom. These manipulatives should always be connected to written symbols and methods.[7]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. All standards figure in a mathematical education and will therefore be eligible for inclusion on the Partnership for Assessment of Readiness for College and Careers (PARCC) assessment. However, the assessments will strongly focus where the standards strongly focus.

In addition to identifying the Major, Supporting and Additional Clusters for each grade, suggestions are given following the table below for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it in ways that foster greater focus and coherence.

[7] See page 19 of the K–8 Publishers' Criteria, www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf.

Key: Major Clusters; Supporting Clusters; Additional Clusters

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

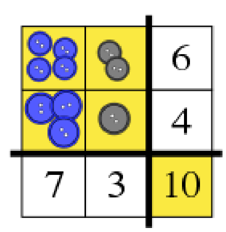
Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

So much is brand new to children in kindergarten that as much as possible, everything throughout the school day should support everything else, as for example when language supports number.

- Even within mathematics itself, understanding, for example, that 18 is ten ones and eight more ones (K.NBT.A.1) requires, but also supports, understanding what it means to combine 10 and 8 or to take apart 18 (K.OA).
- K.MD.B.3 offers a context in which to decompose 10 in more than one way (see K.OA.A.3). For example, given a collection of 10 buttons, children could classify by color and size to answer (K.CC.B.5) questions such as "how many *small* buttons do you have," "how many blue buttons do you have" or "how many large gray buttons do you have?" Such a decomposition of objects can show both that 10 = 7 + 3 and that 10 = 6 + 4. (See figure.)



- Students can count vertices (see K.CC.B.5) as a strategy for recognizing shapes in different orientations (see K.G.A.2) and can use shapes as a setting in which to compare numbers (see K.CC.C.6; e.g., count to see which has more vertices, an octagon or a hexagon — see K.G.B.4).

Examples of Key Advances from Kindergarten to Grade 1

- Grade 1 students gradually come to use sophisticated strategies (such as making ten) that depend on the properties of addition and subtraction; by contrast, kindergarten students determined sums and differences primarily by representing problems in concrete terms. Grade 1 students read and write numbers through 120 and learn the early elements of place value, in particular being able to think of a ten as a unit and understanding that the digits of a two-digit number represent the number of complete tens in that number and the number of remaining ones.
- Grade 1 students will use their understanding of place value and the properties of operations to represent, explain and perform addition and subtraction of two-digit numbers in specified cases.
- In the previous grade, kindergarten students learned about addition and subtraction as ways of finding what happens when collections are combined or separated or when their sizes are changed. Building on this base of meaning, grade 1 students represent and solve a large variety of addition and subtraction problems; that is, word problems and problems set in classroom discussions that involve addition and subtraction scenarios such as putting together, taking apart, comparing, etc. with different quantities in the problem unknown.[8]
- In the previous grade, kindergarten students generally saw equations only when the teacher wrote them on the board; kindergarten students were not expected to write equations themselves. Grade 1 students will write equations for a variety of reasons, such as expressing a decomposition of a number (16 = 9 + 7), expressing a piece of reasoning about numbers (9 + 7 = 9 + 1 + 6 along the way to making ten) or representing a word problem with an unknown (9 + ? = 16). Students use the equal sign appropriately, evaluate the truth of an equation and determine unknown numbers that will make an equation true.

Fluency Expectations or Examples of Culminating Standards

1.OA.C.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true, and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

[8] See Table 2 on page 9 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf, which is based on Table 1 on page 88 of the Common Core State Standards for Mathematics.

This standard relates to fluency when the additions and subtractions in the equations fall within 10, as they do in the italicized examples accompanying the standard.

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ☐ – 3, 6 + 6 = ☐.*

This standard is closely related to fact families; for example, knowing the fact family for 8 + 3 = 11 means being able to find the unknown number in 8 + ? = 11.

1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

Quickly finding 10 more or 10 less than a two-digit number is best thought of as an indicator of whether students have an understanding of place value for two-digit numbers.

Examples of Major Within-Grade Dependencies

- Standard 1.OA.B.3 calls for students to "apply properties of operations" and gives the example "If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known." Similarly, knowing 13 – 3 gives a good starting place for figuring out 13 – 4. Use of properties lets us apply knowledge we have to situations we need to figure out. But *all* of that depends on having some starting places. Standards 1.OA.C.6 and 1.NBT.C.5 are such starting places and are essential building blocks for all of the arithmetic of grade 1. They must therefore be given ample attention early in the year. Though often notated on paper, 1.OA.C.6 and 1.NBT.C.5 as well as 1.NBT.C.6 are essentially *mental* arithmetic knowledge and reasoning.
- 1.NBT.B.2 describes the place value foundations for 1.NBT.B.3 and 1.NBT.C.4. Comparing numbers (1.NBT.A.3) involves thinking about the sizes of tens and ones, and adding two-digit numbers (1.NBT.B.4) involves adding tens with tens and ones with ones, sometimes composing a ten. These ideas and methods rest on an understanding of the place value units (1.NBT.A.2).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- A thorough understanding of how place value language and notation represent number (cluster 1.NBT.A) serves calculation (cluster 1.NBT.B) in many ways — not just pencil-and-paper calculation, but mental calculation as well. It is valuable for purposes of calculation to know that numbers are named so that "twenty-eight" is 20 + 8 and "forty-one" is 40 + 1. That is, the names are designed to make such calculations easy so that we can base calculations like 28 + 41 on it using properties of the operations (1.OA.B.4). This kind of flexible mental arithmetic is a sign of mastery and complements fluency with more algorithmic methods.

- The study of word problems in grade 1 (1.OA.A.1, 1.OA.A.2) can be coordinated with students' growing proficiency with addition and subtraction within 20 (1.OA.C.6) and their growing proficiency with multidigit addition and subtraction (1.NBT).[9]
- Word problems can also be linked to students' growing understanding of properties of addition and the relationship between addition and subtraction. For example, put together/take apart problems with addend unknown can show subtraction as finding an unknown addend.[10]
- Units are a connection between place value (1.NBT) and measurement (1.MD). Working with place value depends on having a sense of the sizes of the base ten units and being able to see a larger unit as composed of smaller units within the system. As measurement develops through the grades, it also depends on having a sense of the sizes of units and being able to see a larger unit as composed of smaller units within the system. Grade 1 is when students first encounter the concept of a tens unit, and it is also when they first encounter the concept of a length unit. In later grades, unit thinking will become important throughout arithmetic, including in the development of multidigit multiplication and division algorithms and the development of fraction concepts and operations.[11]
- Measurement standards 1.MD.A.1 and 1.MD.A.2 together support and provide a context for the 1.OA.A.1 goal of solving subtraction problems that involve comparing. To meet standard 1.MD.A.1, students compare the lengths of two objects by means of a third object, e.g., a length of string that allows a "copy" of the length of one immovable object to be moved to another location to compare with the length of another movable object. When students cannot find the *exact* difference because of the magnitude of the numbers that arise from measurement — as may occur in comparing two students' heights — they may still compare the measurements to know which is greater (1.NBT.B.3). (Grade 2 standard 2.MD.B.6 formalizes this idea on a number line diagram.)
- While students are dealing with the limited precision of only whole and half-hours, they must distinguish the position of the hour hand and connect this to geometry standard 1.G.A.3, partitioning circles into halves and quarters.

[9] See page 13 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.
[10] See page 13 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.
[11] See "Units, a Unifying Idea in Measurement, Fractions, and Base Ten," http://commoncoretools.me/2013/04/19/units-a-unifying-idea/.

- Composing shapes to create a new shape (1.G.A.2) is the spatial analogue of composing numbers to create new numbers. This is also connected to length measurement (1.MD.A.2) since students must visualize an object to be measured as being built up out of equal-sized units (see also 1.G.A.3). Though assembling two congruent right triangles into a rectangle does not use the same facts or reasoning that assembling two 5s into a 10 uses, the idea of looking at how objects in some domain (numbers or shapes) can be combined to make other objects in that domain and looking for new true statements one can make about these combinations is a big idea that is common across mathematics.

Measuring a hallway using students as length units (1.MD.A.2). (The students are idealized as having equal heights.)

Examples of Opportunities for In-Depth Focus

1.NBT.B.2 Grade 1 is students' first encounter with the three linked components of the place value system: base ten units, bundling and unbundling of units, and positional notation. Understanding place value is the foundation of the entire NBT domain.

1.NBT.C.4 Understanding place value is not a final goal on its own; the goal is to use place value understanding and properties of operations to add and subtract (cluster 1.NBT.B). Students learn how standard notation presents and/or records problems for which students have developed mental strategies — adding 10 (1.NBT.C.5) repeatedly and adding 1 repeatedly (counting on, 1.OA.C.6) to a two-digit number — and extends that to adding two arbitrary two-digit numbers (with a result within 100). Being able to represent these additions with materials that show the base ten structure, having a strong mental image and ability with simple versions of these additions, understanding how the notation records these additions, and being able to interpret subtraction in its relation to addition are the foundation for all future arithmetic in elementary school.

1.OA.A.1 There are many distinct elementary addition and subtraction situations; students in grade 1 should work extensively with all of them.[12]

[12] Some situation types need not be mastered until grade 2. See Table 2 on page 9 of the *Progression* document http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practice should be evident throughout mathematics instruction and connected especially to the most important work of the grade. Mathematical tasks (short, long, scaffolded and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

- All work with properties (1.OA.B.3) and with understanding and using place value (e.g., 1.NBT.B.2, 1.NBT.C.4) should be seen as an investigation and use of the structure of our number system and of arithmetic (MP.7). Students' explanations of the properties and reasoning that they used in these contexts are early beginnings of the construction of (brief) logical arguments (MP.3). Examples of brief but excellent arguments at this grade level could include:
    - I know that 7 – 3 equals 4 because 4 + 3 equals 7 (shows 1.OA.B.4 being met).
    - I knew that 8 + 8 = 20 was wrong because 10 + 10 equals 20 and 8 is less than 10.
    - I know that 8 + 7 equals 15 because I know that 8 + 8 equals 16.
- The experience of starting at some number (e.g., 23) and *counting* on 10, and then 10 more, and then 10 more, and then 10 more, and so on, and *hearing* the repetition (e.g., "thirty-three, forty-three, fifty-three, sixty-three") is often a bit of a surprise to students and quite powerful. From the repeated reasoning, they abstract a pattern (MP.8): They describe this in various ways but sometimes say variations on "adding 10 to any number rhymes" and it "changes the counting-by-tens word I use." Thus, content standard 1.NBT.C.4 is being approached by applying standard MP.8 of mathematical practice.
- Students in grade 1 work with some sophisticated addition and subtraction situations (1.OA.A.1), such as "Julie has 8 more apples than Lucy. Julie has 12 apples. How many apples does Lucy have?" The equations 12 – 8 = ? and ? + 8 = 12 are both mathematical models of this situation (MP.4).
- Students working with sums of single-digit numbers (1.OA.C.6) have opportunities to look for and express regularity in repeated reasoning (MP.8). For example, students could be given pairs of addition problems like:[13]

    5 + 6 = ?
    5 + 7 = ?

    3 + 5 = ?
    4 + 5 = ?

    7 + 7 = ?
    8 + 7 = ?

[13] Example adapted from Russell, S.J., D. Schifter and V. Bastable. 2011. *Connecting Arithmetic to Algebra: Strategies for Building Algebraic Thinking in the Elementary Grades.* Portsmouth, NH: Heinemann.

If students are asked to describe what they notice and formulate a general statement, they can come to a conclusion that increasing one of the addends by 1 increases the sum by 1. In later grades, this can be seen as a special case of properties, $a + (b + 1) = (a + b) + 1$ or $(a + 1) + b = (a + b) + 1$.

A note on manipulatives in grades K–2: Manipulatives such as physical models of hundreds, tens and ones are an important part of the K–2 classroom. These manipulatives should always be connected to written symbols and methods.[14]

Content Emphases by Cluster

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

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In addition to identifying the Major, Supporting and Additional Clusters for each grade, suggestions are given following the table below for ways to connect the Supporting to the Major Clusters of the grade. Thus, rather than suggesting even inadvertently that some material not be taught, there is direct advice for teaching it in ways that foster greater focus and coherence.

[14] See page 19 of the K–8 Publishers' Criteria, www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf.

Key: Major Clusters; Supporting Clusters; Additional Clusters

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction. (Major)
- Understand and apply properties of operations and the relationship between addition and subtraction. (Major)
- Add and subtract within 20. (Major)
- Work with addition and subtraction equations. (Major)

Number and Operations in Base Ten

- Extend the counting sequence. (Major)
- Understand place value. (Major)
- Use place value understanding and properties of operations to add and subtract. (Major)

Measurement and Data

- Measure lengths indirectly and by iterating length units. (Major)
- Tell and write time. (Additional)
- Represent and interpret data. (Supporting)

Geometry

- Reason with shapes and their attributes. (Additional)

Examples of Linking Supporting Clusters to the Major Work of the Grade

- When students work with organizing, representing and interpreting data, the process includes practicing using numbers and adding and subtracting to answer questions about the data (see the part of 1.MD.C.4 after the semicolon, and see the K–5 MD *Progression* document, especially Table 1 on page 4 and the discussion of categorical data on pages 5 and 6).[15]
- Telling and writing time on digital clocks (1.MD.B.3) is a context in which one can practice reading numbers (1.NBT.A.1), a kind of "application," but no more. Relating those times to *meanings* — events during a day — is not part of 1.MD.B.3, but making sense of what one is doing (MP.1) and contextualizing (MP.2) are essential elements of good mathematical practice and should be part of the instructional foreground at all times.

[15] See http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf.

Examples of Key Advances from Grade 1 to Grade 2

- Where grade 1 students worked within 100, grade 2 students will read and write numbers through 1,000, extending their understanding of place value to include units of hundreds.
- Similarly, grade 2 students use their understanding of place value to add and subtract within 1,000 (e.g., 237 + 616 or 822 – 237). They can explain what they are doing as they add and subtract. They become fluent in addition and subtraction within 100.
- For word problems, students extend their ability by solving two-step problems using addition, subtraction or both operations. They also master more advanced one-step addition and subtraction problems in this grade (such as take from with start unknown).[16]
- Students use standard units of measure and appropriate measurement tools. They understand basic properties of linear (length/distance) measurement, such as the fact that the smaller the unit, the more iterations will be needed to cover a given length.

Fluency Expectations or Examples of Culminating Standards

2.OA.B.2 Fluently add and subtract within 20 using mental strategies.[17] By end of Grade 2, know from memory all sums of two one-digit numbers.

2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Students can also show their fluency using an efficient, general algorithm.[18]

2.NBT.A.2 Count within 1000; skip-count by 5s, 10s, and 100s.

Fluency here is helpful because skip counting is sometimes a strategy for adding or subtracting.

2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Students who struggle to read a three-digit number may not grasp place value.

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[16] See Table 2 on page 9 of the *Progression* document for K–5 Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf, which is based on Table 1 on page 88 of Common Core State Standards for Mathematics.

[17] See standard 1.OA.C.6 for a list of mental strategies. *(Footnote in Common Core State Standards for Mathematics.)*

[18] For the difference between a computation *strategy* and a computation *algorithm*, see the glossary of the standards (page 85, under the letter "c" for "computation").

2.NBT.B.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.MD.A.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Sufficient practice is required to measure accurately and reasonably quickly.

Examples of Major Within-Grade Dependencies

- Understanding place value (cluster 2.NBT.A) is the foundation for using place value understanding and the properties of operations to add and subtract (cluster 2.NBT.B). (Mastery of the two clusters can grow over time in tandem with one another.) Adding and subtracting within 1,000 (2.NBT.B.7) involves adding or subtracting hundreds with hundreds, tens with tens and ones with ones, sometimes composing or decomposing tens or hundreds. These ideas and methods rest on an understanding of the place value units (2.NBT.A.1, building on 1.NBT.A.2).
- Knowing single-digit sums from memory (2.OA.B.2) is the basis for adding and subtracting multidigit numbers fluently and efficiently in general (cluster 2.NBT.B).

Examples of Opportunities for Connections among Standards, Clusters or Domains

- Representing whole numbers as lengths (2.MD.B.6) and comparing measurements (2.MD.4) can build a robust and flexible model for fluent subtraction (2.OA.A.1). For example, a good way to see the "distance" from 6 to 20 is to see the distance from 6 to 10 joined with the distance from 10 to 20.
- Problems involving dollars, dimes and pennies (2.MD.C.8) should be connected with the place value learning of 100s, 10s and 1s (2.NBT.A.1). Though the notation is different, a dollar is 100 cents or a "bundle" of 10 dimes, each of which is a "bundle" of 10 pennies. Work with dollars, dimes and pennies (without the notation) can support methods of whole-number addition (e.g., dimes are added to dimes). Addition that is appropriate with whole numbers can be explored in the new notation of money contexts (though fluency with that notation is *not* a standard at this grade).
- Students' work with addition and subtraction word problems (2.OA.A.1) can be coordinated with their growing skill in multidigit addition and subtraction (2.OA.B.2, cluster 2.NBT.B).
- Work with nickels (2.MD.C.8) and with telling time to the nearest five minutes on analog clocks (2.MD.C.7) should be taken together with counting by 5s (2.NBT.A.2) as contexts for gaining familiarity with groups of 5 (2.OA.C.4). Recognizing time by seeing the minute hand at 3 and *knowing* that is 15 minutes; recognizing three nickels as 15¢; and seeing the 15-ness of a 3-by-5 rectangular array held in any position at all (including with neither base horizontal) will prepare for understanding, in grade 3, what the new operation of multiplication means.
- The number line (2.MD.B.6) connects numbers, lengths and units. The number line increases in prominence across the grades.

Examples of Opportunities for In-Depth Focus

2.OA.A.1 Using situations (from word problems, from classroom events or experiences, and from discovered mathematical patterns) as a source of problems can help students make sense of and contextualize the operations they are learning. There is a tremendous variety of basic situation types in addition and subtraction.[19]

2.NBT.B.7 It takes substantial time throughout the year for students to extend addition and subtraction to 1,000, connecting steps in the computation to what they know about place value and properties of operations.

Examples of Opportunities for Connecting Mathematical Content and Mathematical Practice

Mathematical practice should be evident throughout mathematics instruction and connected especially to the most important work of the grade. Mathematical tasks (short, long, scaffolded and unscaffolded) are an important opportunity to connect content and practices. Some brief examples of how the content of this grade might be connected to the Standards for Mathematical Practice follow.

- *Complements:* Students can generalize the kindergarten idea of finding the number that makes 10 when added to a given number (K.OA.A.4). For example, students can find the number that makes 30 when added to 12. This approach — learning a set of "content" facts and recognizing/building a structure behind them (MP.7) through repeated use in varied contexts (MP.8) — makes very appropriate use of good mathematical practice.
- Grade 2 students make frequent use of objects that remain appropriate tools (MP.5) for a lifetime: rulers, clocks, coins and the number line (essentially an abstract ruler or measurement scale).
- *Knowing 5:* Standard MP.5 is about not just the ability to *use* tools, but the ability to *choose* the appropriate tool for a task. At this stage, because students are just beginning to use a variety of tools, their utility may seem both obvious and fixed to a task: a ruler measures length, for example, and students have little to make choices about. But coming to understand the *significance* of counting by 5 — the usefulness of that litany and the situations in which it appears — may involve, for some students, a choice of which of several images (nickels, hands, telling time) is most clarifying and salient to them. Generating the abstraction — the litany 0, 5, 10, 15, etc. — may also be aided by experiences in the various domains, including the recognition that one sequence of numbers is common to all of them. That sequence of number names expresses the regularity (MP.8) of a calculation (counting five more) that recurs in many contexts.

[19] See Table 2 on page 9 of the *Progression* document for K–5 Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf, as well as www.achievethecore.org/page/258/representing-and-solving-addition-and-subtraction-problems.

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Key: ■ Major Clusters; ▫ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking

- ■ Represent and solve problems involving addition and subtraction.
- ■ Add and subtract within 20.
- ▫ Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- ■ Understand place value.
- ■ Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- ■ Measure and estimate lengths in standard units.
- ■ Relate addition and subtraction to length.
- ▫ Work with time and money.
- ▫ Represent and interpret data.

Geometry

- ○ Reason with shapes and their attributes.

Examples of Linking Supporting Clusters to the Major Work of the Grade

- When students work with time and money (cluster 2.MD.C), their work with dollars, dimes and pennies should support their understanding and skill in place value (2.NBT). Their work with nickels, with telling time to the nearest five minutes on analog clocks, with counting by 5s (2.NBT.A.2), and with arrays of five rows and/or five columns (cluster 2.OA.C) should be taken together.
- In cluster 2.MD.D, "Represent and interpret data," standard 2.MD.D.10 particularly represents an opportunity to link to the major work of grade 2. Picture graphs and bar graphs can add variety as contexts for solving addition and subtraction problems. The language in 2.MD.D.10 mentions word problems (2.OA) explicitly. See the *Progression* document for K–5 Measurement and Data for more on the connections between data work and arithmetic in the early grades.[21]
- Without adding greater meaning or depth, 2.MD.D.9 is a potential context for 2.MD.A.1 and gives students a first taste of visual comparison of numerical information (though the fact that this numerical information was derived from length makes the representation more about scaling the information than about visualizing it).

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[21] See http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf.